Purushottam School of Engineering and Technology, Rourkela

Lectures notes
On

THERMAL ENGINEERING-II (MET 403)
4th SEM MECHANICAL

Department of Mechanical Engg.

Prepared by:-
Mr.Priyabrata Jena
(Lecturer)

THERMAL ENGINEERING-II

Name of the Course: Diploma in MECHANICAL ENGINEERING			
Course code:	MET 403	Semester	4th
Total Period:	60	Examination	3 hrs
Theory periods:	4 P/W	Class Test:	20
Tutorial:		Teacher’s Assessment:	10
Maximum marks:	100	End Semester Examination:	70

Course Objectives:

- Students will develop an ability towards
- Comprehending major theoretical cyclic processes using vapor and gas as working substances and computing work done and efficiencies thereof.
- Comprehending heat transfer modes and computing heat transferred through conduction, convection and radiation from simple structures.
- Comprehending refrigeration cycles in practice and computing coefficient ofm performance and efficiencies.

Chapter	Topics	Contents	Hours
1.	**Vapor Power Cycles**		12
	1.1	Steam power plant lay out	
	1.2	Steam power plant cycle	
	1.3	Carnot vapor cycle	
	1.4	Rankine vapor cycle	
	1.5	Modifications to Rankine vapor cycles	
	1.6	Qualities of ideal working fluid for vapor power cycle	
	1.7	Binary vapor cycles	
2	**Gas Power cycles**		12
	2.1	Concept of IC Engine	
	2.2	Otto cycle	
	2.3	Diesel cycle	
	2.4	Dual cycle	
	2.5	Comparison of Otto, Diesel and dual cycles	
	2.6	2S and 4S engines and differences thereof	
3	**Fuels and Combustion**		10
	3.1	Hydrocarbon fuels	
	3.2	Combustion reactions (Explanation only), concept of stoichiometric combustion, complete combustion and incomplete combustion	
	3.3	Enthalpy of formation, enthalpy of reaction	
	3.4	Heating values for fuels	
	3.5	Quality of IC Engine fuels: Octane Number and Cetane number	
4	**Heat Transfer**		16
	4.1	Modes of heat transfer	
	4.2	Fourier law of heat conduction, thermal conductivity	
	4.3	Steady state heat conduction in solids (Plane wall, hollow cylinder, hollow sphere)	
	4.4	Convective heat transfer, Newton’s law of cooling	
	4.5	Radiation heat transfer, Stefan Boltzman Law	
	4.6	Theories of radiation: Maxwell’s theory, Max Planck’s theory; Black body radiation	
	4.7	Surface absorption, reflection and transmission	
	4.8	Kirchoff’s law relating to spectral emissive power to absoptivity	
	4.9	Heat exchangers: concept, application and classification	
5	**Refrigeration cycles**		10
	5.1	Concept of refrigerators and heat pumps	
	5.2	Reversed Carnot cycle and its limitations	
	5.3	Ideal vapor compression refrigeration cycle	

5.4 Actual vapor compression refrigeration cycle
5.5 Gas refrigeration cycle

Learning Resources:

Text Books:	Engineering Thermodynamics,	P. Chattopadhyay
	Thermal Engineering:	Mahesh M Rathore

MET -303 THERMAL ENGINNERING-1

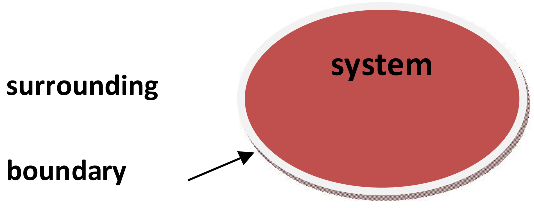
CHAPTER 1:

CONCEPTS AND TERMINOLOGY

- **Thermodynamics**
  It is defined as the science of heat energy transfer and its effect on physical property of the substance.
  OR
  It may be defined as the science which deals with the conversion of heat into mechanical work or energy by using a suitable medium.
- **Thermodynamic System**
  System: A system is defined as any quantity of matter or a region in space having certain volume upon which our attention is concerned in analysis of problem.

  Surrounding: Anything external to the system constitute as surrounding.

  Boundary: System is separated from the surrounding by system boundary. This boundary may be fixed or movable.



system are classified into three types :-

- Open system
- Closed system
- Isolated system

Open System
It is also known as *flow system.* Open system is one in which both mass and energy crosses the boundary. Open system is also called control volume. Ex- reciprocating air compressor, turbine, pump etc.

Closed System
It is also known as *non-flow system*. In this system the mass within the boundary remains constant only energy interaction takes place with respect to the surrounding. Ex – Cylinder piston arrangement, Tea kettle.

Isolated System
An isolated system is one in which there is no interaction between the system and surrounding. There is no mass and energy transfer across the system. Ex- Universe, thremoflask etc.

MACROSCOPIC AND MICROSCOPIC APPROACH

Study of thermodynamics is done by two different approaches.

- **Macroscopic approach**: The term macroscopic is used in regard to larger units which is visible to the naked eye. In macroscopic approach certain quantity of matter is considered without taking into consideration the events occurring at molecular level. In other words macroscopic approach is concerned with overall behaviour of matter. This type of study is also known as *classical thermodynamics*.
- **Microscopic approach**: In microscopic approach matter is considered to be composed of tiny particles called molecules and study of each particle having a certain position, velocity and energy at a given instant is considered such a study is also called as *Statistical thermodynamics.*

CONCEPT OF CONTINUUM

The system is regarded as a continuum i.e. the system is assumed to contain continuous distribution of matter. Thus, from the continuum point of view, the matter is seen as being distributed through space and treats the substance as being continuous disregarding the action of individual molecules. There are no voids and values of action of many molecules and atoms.

THERMODYNAMIC PROPERTY

- **PROPERTY-**A thermodynamic property refers to the characteristics by which the physical condition or state of a system can be described such as pressure, volume, temperature etc. & such characteristics are called properties of a system.
- **PRESSURE-**Pressure is defined as force per unit area.

  Units of pressure are as follows In S.I Pascal (Pa) and 1 Pa= 1N/m^2

  1 Bar= 10^5 N/m^2 = 100 KPa

  1 ATM=760mm of Hg or 1.013 bar or 101.325KPa

- **TEMPERATURE-**The temperature is a thermal state of a body which determines the hotness or coldness of a body. The temperature of a body is proportional to the stored molecular energy i.e. the average molecular kinetic energy of the molecules in a system.
  Units of temperature are degree Celsius or Kelvin.

Intensive and Extensive Property:

- Intensive property: The properties which are independent of mass of the system are known as intensive properties. Its value remains the same whether one considers the whole system or only a part of it. The intensive property includes pressure, temperature, specific volume, specific energy, specific density etc.

- Extensive property: the property which depends upon mass of the system are known as extensive property. The extensive properties include volume, energy, enthalpy, entropy etc.

State: The condition of physical existence of a system at any instant of time is called state.

Thermodynamic Processes:

When any property of a system changes, there is a change in state and the system is then said to have undergo a thermodynamic process.

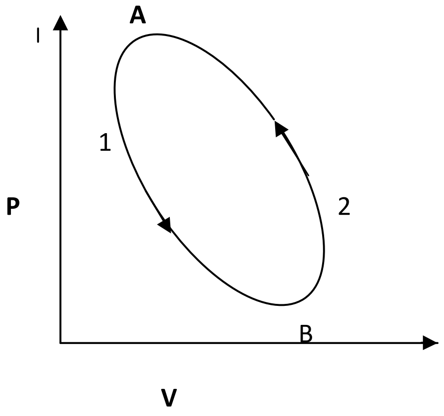
The commonly used processes are:

1. Isochoric Process – The process which takes place at constant volume is said to be isochoric process.
2. Isobaric Process – The process which takes place at constant pressure is said to undergo an isobaric process.
3. Isothermal Process- The process which takes place at constant temperature is said to undergo an isothermal process.
4. Adiabatic Process- The process where there is no heat transfer between the system and the surrounding. The reversible adiabatic process is known as isentropic process.

   The other processes are polytrophic process, throttling process, free expansion process and hyperbolic process.

Thermodynamic Cycle:

When a process is performed in such a way that the final state is identical with the initial state, it is then known as a thermodynamic cycle or cyclic process.



In the fig above:

A-1-B and **A-2-B** represents *process*

Whereas **A-1-B-2-A** represent a *thermodynamic cycle*.

THERMODYNAMIC EQULIBRIUM:

A system is said to be in thermodynamic equilibrium when no change in any macroscopic property is registered, if the system is isolated from its surrounding.

Thermodynamics mainly studies the properties of physical system that are found in equilibrium state.

A system will be said to be in thermodynamic equilibrium if the following three conditions of equilibrium is satisfied.

a) Mechanical Equilibrium
b) Chemical Equilibrium
c) Thermal Equilibrium

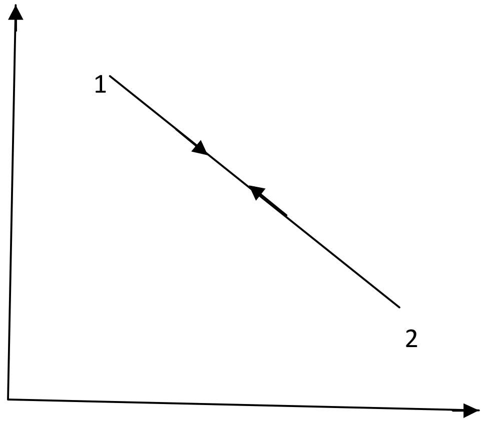
Mechanical Equilibrium- when there is no unbalanced force on any part of the system or in-between the system and surrounding then the system is said to be in mechanical equilibrium. For example if the pressure is not uniform throughout the system, then internal changes in the state of the system will take place until the mechanical equilibrium is reached.

Chemical Equilibrium- when there is no chemical reaction or transfer of matter from one part of the system to another such as diffusion or solution, then the system is said to exists in a state of chemical equilibrium.

Thermal Equilibrium- when there is no temperature difference between the parts of the system or between the system and the surrounding, it is then said to be in thermal equilibrium.

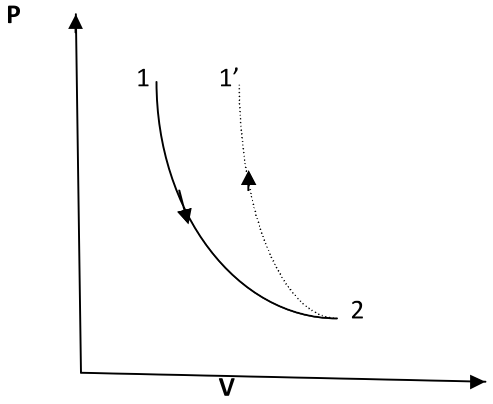
REVERSIBLE PROCESS:

A process which can be reversed in direction and the system retraces the same continuous series of equilibrium states it is said to be reversible process. A reversible process should be carried out with absolute slowness, so that the system will be always in equilibrium. In actual practise a reversible process cannot be attained, but it can be approximated as a closely as a possible. For example a gas confined in a cylinder with a well lubricated piston can be made to undergo a reversible process by pushing or pulling the piston in slow motion.



IRREEVERSIBLE PROCESS

A process in which the system passes through a sequences non-equilibrium state i.e. The property such as pressure, volume, temperature is not uniform throughout the system it is known as an irreversible process. This process will not retrace the reverse path to restore the original state. The heat transfer by convection, combustion of air and fuel etc are few examples of irreversible process.



QUASI-STATIC PROCESS

The word quasi means *almost.* This process is a succession of equilibrium states and infinite slowness is the characteristic feature of quasi-static process. A quasi-static process is also called as reversible process, the basic difference is that in a quasi static process not all the point but almost major points is in equilibrium condition.

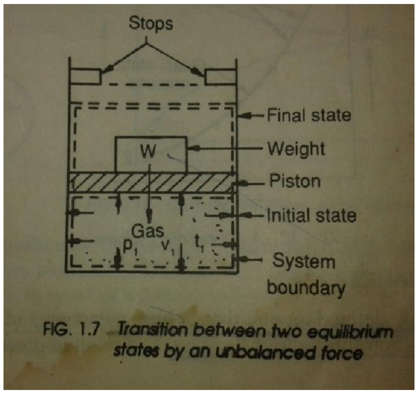


Fig 1

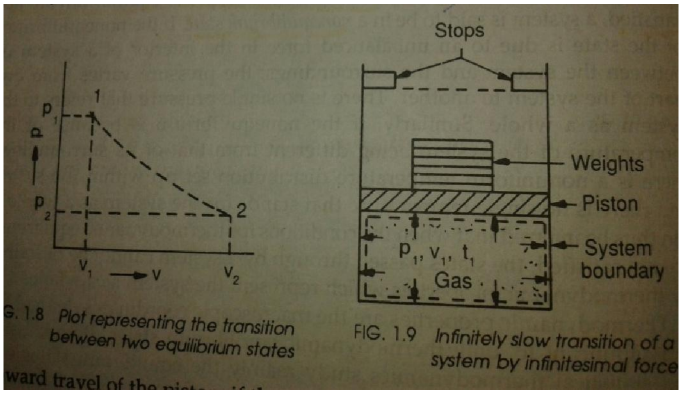


Fig 2

Let us consider a system of gas contained in a cylinder as shown in fig 1 . The system is initially an equilibrium state. The weight W on the piston just balances the upward force exerted by the gas. If the weight is removed there will be an unbalanced force between the system and the surrounding, and the piston will move upward till it hits the stops .the system will be again in a equilibrium state but if the same process as shown if fig 2 is done by slowing removing very small pieces of weight one by one then the piston will move upward slowly thus the system will be in equilibrium.

CHAPTER 2:

ENERGY AND WORK TRANSFER

A closed system interacts with the surrounding by energy transfer and this energy transfer takes place in two ways i.e. *Work transfer* and *Heat transfer.*

Heat and work are the main mode of energy transfer and there are certain similarities and differences between heat and work.

- The heat and work are boundary phenomena. They are observed at the boundary of the system.
- When a system undergoes a change in state, heat transfer or work done may occur.
- Heat and work are path function and depends upon the process. Hence they are not thermodynamic property and are inexact differential.
- Work is said to be *high grade energy* and heat low *grade energy*. The complete conversion of low grade energy into high grade energy is impossible.

Work Transfer

The action of a force on a moving body is identified as work. For the work transfer the system has to be such selected that its boundary just move. There cannot be work transfer in a closed system, without moving the system boundaries. In a cylinder piston arrangement the top of the system is moving system boundary and the work is transferred by the movement of the piston.

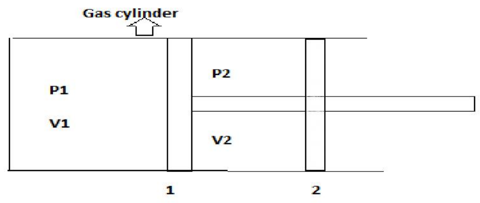
Work done by the system is considered to be *positive* and work done on the system is taken as *negative.*

Work done= force X displacement

Unit of work: Newton-meter (N-m) or Joule(J)

The rate at which work is done upon or by the system is known as *power.* The unit of power is J/s or watt.

PdV work or Displacement Work



Let us consider a gas in the cylinder as shown in the fig above. Let the system initially be at pressure P1 and volume V1. The system is in thermodynamic equilibrium , the piston is the system boundary which moves due to gas pressure. Let the piston move out to a new final position 2 which is also in thermodynamic equilibrium specified by pressure P2 and volume V2. When the piston moves an infinitesimal distance ***dl*** if ***a*** be the area of the piston.

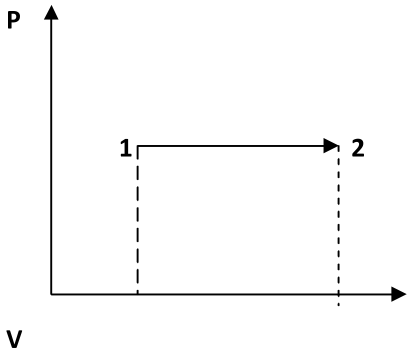
The force F acting on the piston will be
F= p X a
The amount of work done by the gas on the piston will be
dW = F. dl = p X a X dl = pdV
where dV= a X dl
when the piston moves out from position 1 to position 2 then the amount of work done b the system will be
$W_{1-2}= \int_{v1}^{v2} pdv$
The above equation represents the displacement work.

Displacement work applied to different thermodynamic process

1. **Isobaric process**

$W_{1-2}= \int_{v1}^{v2} pdv$

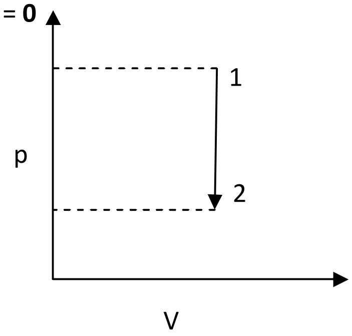
$= p(v_2 - v_1)$



2. **Isochoric process**

$W_{1-2}= \int_{v1}^{v2} pdv$

= **0**



3. Isothermal process

In this process pv= constant

$pV = P_1V_1 = C$

$P= P_1V_1/ V$

$W_{1-2}= \int^{v2}_{v1} pdv$

$W_{1-2}= p_1V_1\int^{v2}_{v1} dV/V$

= $\mathbf{p_1V_1 \ln p_1/p_2}$

4. Polytrophic process

The process in which expansion and contraction takes lace according to the law pV^n=C

$pV^n = p_1v_1^n = p_2V_2^n = C$

$W_{1-2}= \int^{v2}_{v1} pdv$

$= \int^{v2}_{v1} (p_1V_1^n/ V^n). dV$

$= p_1V_1^n[V^{-n+1}/ -n+1]_{v1}^{v2}$

$= [p_2V_2^n *V_2^{1-n} – p_1V_1^n *V_1^{1-n}]/ (1-n)$

= $\mathbf{(p_1V_1 – p_2V_2) / (n-1)}$

HEAT TRANSFER

Heat is defined as the form of energy that is transferred across a boundary by virtue of temperature difference between the system and the surroundings.

It the heat flows into the system or the system receives heat then heat transfer Q is taken as *positive* and if heat is rejected from the system then Q is taken as *negative.*

The heat transfer takes place by three different modes

1. Conduction: the transfer of heat between two bodies in direct contact is called conduction. It is a process of heat transfer from one particle of a body to another in the direction of fall of temperature. For example heat transfer through solids is by conduction.
2. Convection : The process of heat transfer from one particle to another by convection currents i.e. transfer of heat between the wall and fluid system in motion. In this case, the particles of the body move relative to each other.
3. Radiation : Heat transfer between two bodies separated by empty space or gases through electromagnetic waves is radiation.

Sensible heat: The heat required for change from liquid state to vapourisation/boiling point is called sensible heat. It is the amount of heat absorbed by one kg of water, when heated at a constant pressure, from the freezing point (0^0C) to the temperature of formation of steam.

Latent heat: It is the amount of heat absorbed to evaporate one kg of water at its boiling point without change of temperature.

Specific Heat : The amount of heat required to raise the temperature of unit mass of a substance through one degree is known as specific heat.

The unit of specific heat is KJ/kg K

Mathematically heat required to raise the temperature of a body is

$Q = m\ C(T_2-T_1)$ in kJ

Where, m= mass of the substance in kg

C= specific heat in KJ/Kg K
T_1= initial temperature in degree Celsius or Kelvin
T_2= final temperature in degree Celsius or Kelvin

Specific heat at constant volume (Cv) : It is defined as amount of heat required to raise the temperature of a unit mass of a gas by one degree at constant volume.

Specific heat at constant pressure (Cp) : It is defined as amount of heat required to raise the temperature of a unit mass of a gas by one degree at constant pressure

ENERGY:

The energy is defined as the capacity to do work. In broad sense energy is classified as *stored energy* and *transient energy.*

The energy that remains within the system boundary is called ***stored energy*** e.g. potential energy, kinetic energy and internal energy.

The energy which crosses the system boundary is known as energy in transition e.g. heat, work, electricity etc.

DIFFERENT FORM OF STORED ENERGY

1. POTENTIAL ENERGY-The energy posed by a body by a virtue of its position or state of rest is known as potential energy
   P.E=W X h=mgh
   W=weight of the body in N
   M=mass of the body in kg
   g=acceleration due to gravity
   h=height in meter

2. KINETIC ENERGY-The energy posed by a body by virtue of its motion.

Mathematically kinetic energy,

K.E=1/2 mv^2

V=velocity of the body

3. INTERNAL ENERGY –The energy posed by a body or a system by virtue of its intermolecular arrangement and motions of molecules. The change in temperature causes the change in internal energy. It is usually denoted by U.

The sum of the above three energies is the total energy of the system

E=P.E+ K.E+ U

But when the system is stationary and the effect of gravity is neglected then P.E=0 and K.E=0. Thus

E = U

I.E the total energy is equal to the total energy of system.

CHAPTER 3.0

THERMAL ENGINEERING

FIRST LAW OF THERMODYNAMIC

Introduction

Heat and work are different forms of the same entity called energy. Energy is always conserved. Energy may enter a system as heat and leave as work and vice-versa.

Energy has two forms-transit energy and stored energy.

The internal energy is the stored energy. Whenever heat and work enters a system, stored energy increases and when heat & work leaves the system stored energy decreases.

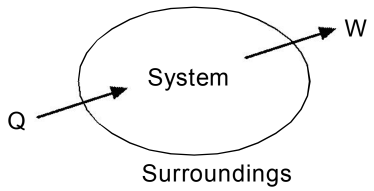
3.1 First law of thermodynamics

Whenever heat is absorbed by a system it goes to increases its internal energy plus to do some external work (Pdv work) i.e.

$$Q = \Delta E + W$$

Where Q is the energy entering a system, ΔE increase in internal energy, W – producing some external work.

$$\delta Q = dE + Pdv$$

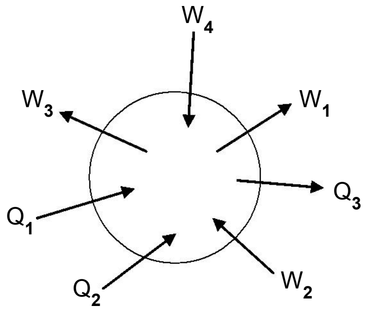


Sometimes more than two energy transfers, so it becomes.

$$Q_1 + Q_2 - Q_3 = \Delta E + W_1 - W_2 + W_3 - W_4$$

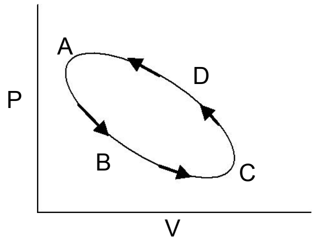
Sign convention

It will be '+Q' if heat goes into the system and '-Q' if heat goes out of the system +W when it is done by the system and –w is done on the system.



Cyclic process

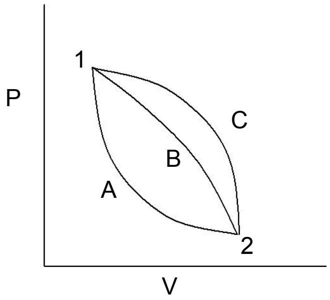
For a cyclic process, the work done is the area enclosed by the PV curve.



Clock wise = + W
anticlockwise = - W

3.2 Energy as a system property

Let a system changes from state 1 to state 2 via path A path B and Path C, which as follows



Applying first law of thermodynamics to path A.

$$Q_A = \Delta E_A + W_A$$

For path B $Q_B = \Delta E_B + W_B$

If process A & B form a complete cycle

$$\sum Q = \sum W$$

$$(Q_A + Q_B) = \Delta E_A + E_B + \left(W_A + W_B\right)$$

$$\sum Q = \Delta E_A + E_B + \sum W$$

$$\Rightarrow \Delta E_A = -\Delta E_B$$

Similarly $\Delta E_A = -\Delta E_B$

$$\Rightarrow \Delta E_B = \Delta E_C$$

So it is independent of path hence a property extensive or in nature.

Different forms of stored energy

Energy can be store in a system by two modes.

(i) Macroscopic mode

(ii) Microscopic mode

(i) Macroscopic mode

In this mode, the mode of stored energy stored in two forms,

$$E_{KE} = \left(\frac{1}{2}\right)MV^2$$

$$E_{PE} = mgz$$

(ii) Microscopic mode

This mode of stored energy refers to energy stored in molecular and atomic structure. Hence it is called molecular internal energy on simply internal energy. Then including

1. Translational KE
2. Rotational KE
3. Vibration energy
4. Electronic energy
5. Chemical energy
6. Nuclear energy

$$L = L_{translation} + L_{rotational} + L_{vibrtion} + L_{electronic} + L_{chemical} + L_{nuclear}$$

Total energy

$$E = E_{KE} + E_{PE} + U$$

In absence of motion, gravity $E_{KE}, E_{PE} = 0$

$$\text{So} E = U$$

$$Q = \Delta U + \int Pdv$$

3.3 First law for a closed system undergoing a cyclic process.

3.5 Enthalpy concept

Enthalpy is a state property of a system. It is denoted by

H = U + Pv

It is a point function and an intensive property.

Specific enthalpy is given by

$$h = u + \frac{1}{m} PV = u + Pv$$

3.6 First law for a steady flow process

Steady flow process

A flow process is the one in which a fluid enters the system and then leaves if after a work interaction.

The mass flow rate and energy flow rate across the system boundary are constant.

Important terms

Flow work – whenever a certain amount of mass enters a system, an amount of work is required to push the mass into the system and out of it to maintain the continuity of flow.

i.e. Flow work = PV

Control Volume

For computation of mass and energy notes during a flow process, it is convenient to focus attention upon a certain fixed region in space called control volume.

Control surface

The boundary line defining the control volume is called control surface.

Stored energy of a system in a flow process

During a steady state flow, there is neither any accumulation of mass nor energy.

$$(\text{Mass flow rate})_{in} = (\text{Mass flow rate})_{out}$$

$$\sum \text{Energy}_{in} = \text{Energy}_{out}$$

Now, the total energy of a fluid at any section of the control volume.

$$E = U + \frac{1}{2}(MV^2) + mgz$$

$$e = u + \frac{1}{2}V^2 + gz$$

STEADY FLOW ENERGY EQUATION

As all energy is conserved

$$\Sigma \text{Energy}_{\text{in}} + \text{Heat flux} = \Sigma \text{Energy}_{\text{in}} + \text{Work}_{\text{output}}$$

i.e.

$$U_1 + \frac{1}{2}mV_1^2 + mgz_1 + P_1V_1 + Q$$

$$= U_2 + \frac{1}{2}mV_2^2 + mgz_2 + P_2V_2 + W$$

$$\left[h_1 + \frac{1}{2}mV_1^2 + mgz_1 + Q\right] + \left[h_2 + \frac{1}{2}mV_2^2 + mgz_2 + W\right]$$

On the basis of per unit mass flow rate

$$h_1 + \frac{V_1^2}{2} + gz_1 + Q/m$$

$$h_2 + \frac{V_2^2}{2} + gz_2 + W/m$$

$$\boxed{\text{or } h_1 + \frac{V_1^2}{2} + gz_1 + Q = h_2 + \frac{V_2^2}{2} + gz_2 + W}$$

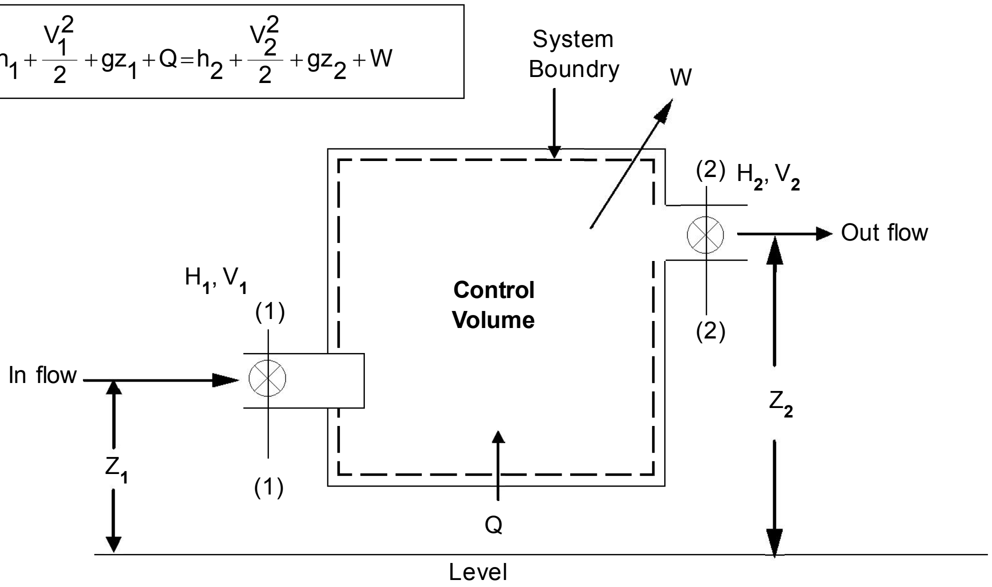


Fig- Steady flow

NOZZLE

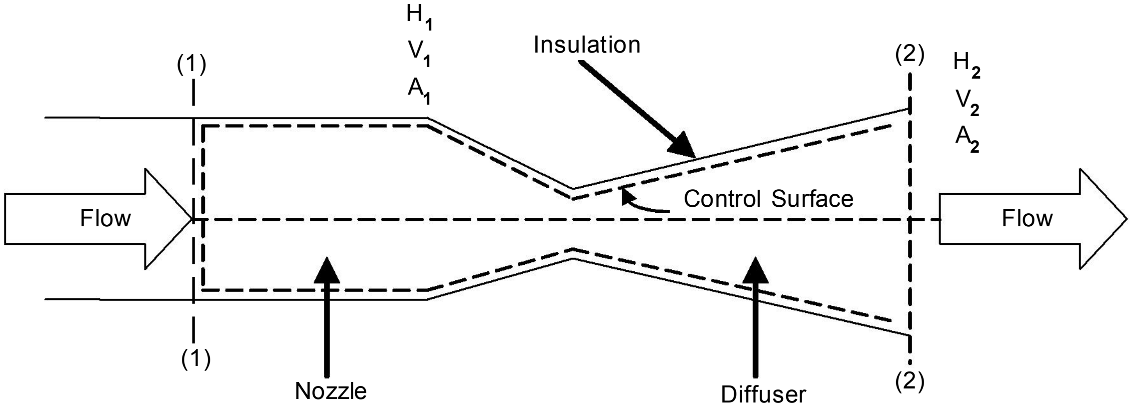


Fig- A Steady flow process through a nozzle.

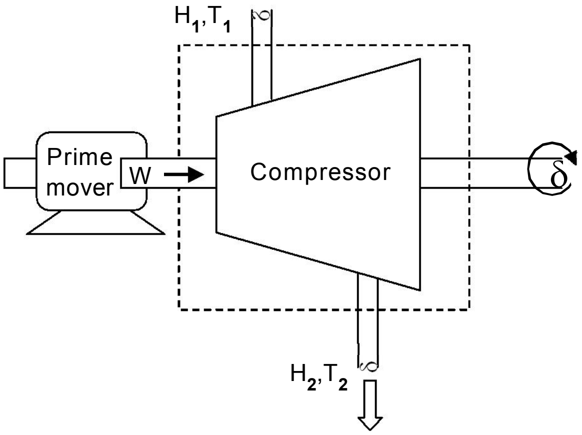
A nozzle is a device used to throttle a fluid whereupon its pressure energy is converted into kinetic energy. The enthalpy of fluid decreases as the velocity of the fluid increases because of a higher fluid velocity at the nozzle outlet, a nozzle is harnessed to gain a high thrust in rockets and jet engines and drive impulse type steam and gas turbines.

Compressor

A compressor compresses air or a gas by harnessing external work fed from a prime mover. The increase in the gas pressure is accompanied by the temperature rise. If the compressor is perfectly insulated and the compression is adiabatic then it requires the minor work input to increase the gas pressure. All the generated heat of compression is expanded to compresses the gas as no heat is allowed to escape.

So Q = 0

Mass flow route of the gas = M and $V_1 = V_2$



Z1 = Z2,

By applying SFEE

$h_1 = h_2 + (-W_C)$

(-) sign before Wc refers to the work done on the gas (system)

$W_C = M(h_2 - h_1)$

$= MC_P (T_2 - T_1)$

Example

An air compressor compresses air from $0.1MP_a$ / 300K to 1 MP_a. The compressor casing is well insulated, yet there is a heat loss to the surrounding to the extent of 5% of the compressor work.

Determine air temp at outlet and power input given

$V_1 = 40m/s$, $V_2 = 100m/s$, $A_1 = 100cm^2$, $A_2 = 20cm^2$, $C_P = 10^3 J\ Kg^{-1}\ K^{-1}$

Solution

$P_2V_2 = RT_2$ or $T_2 = P_2\ {}^{V_2}\!/_R$, V_2 - Specific volume.

We have $m_1 = m_2$

$$\frac{a_1V_1}{v_1} = \frac{a_2V_2}{v_2}$$

$$P_1v_1 = RT_1$$

$$v_1 = \frac{RT_1}{P_1} = \frac{287 \times 300}{0.1 \times 10^6}$$

$$= 0.861 m^3\ kg^{-1}$$

$$\text{and } v_2 = \frac{a_2v_2}{a_1v_1} \times V_1 = \left(\frac{20}{100}\right)\left(\frac{100}{40}\right) \times 0.861$$

$$= 0.4305\, m^3\ kg^{-1}$$

$$T_2 = P_2v_2 / R$$

$$= \frac{1 \times 10^6 \times 0.4305}{287} = 1500K$$

$$Z_1 = Z_2$$

$$Q = 5\,\%\,W_C$$

$-Q$ as rejected

$-W\ ^{\cap}$ W.D on the system

$$H_1 + \frac{1}{2}MV_1^2 + (-Q)$$

$$= H_2 + \frac{1}{2}MV_2^2 + (W_C)$$

$$-0.5W_C + W_C$$

$$= M(h_2 - h_1) + m\left(V_2^2 - V_1^2\right)$$

$$0.95W_C = 0.4646\,x\,C_P\left[(T_2 - T_1) + \frac{1}{2}(V_2^2 - V_1^2)\right]$$

$$m = \frac{A_1V_1}{V_1}$$

$$= \left(100\,x\,10^4\,m^2\right)x\,\frac{40m^5-1}{0.861} = 0.46457kg5^{-1}$$

$$W_C$$

$$= 0.4646\,x\left[10^3\,(1500-300) + \frac{1}{2}\left(100^2 - 40^2\right)\right]$$

$$= 59889\,/\,7w$$

$$= 588.9kw\ \ (Ans)$$

Nozzle

There is no work output : W = O

No heat influx or escape Q = 0

For a horizontal disposition

$$Z_1 = Z_2$$

And so the SFEE applied to the nozzle boils down to

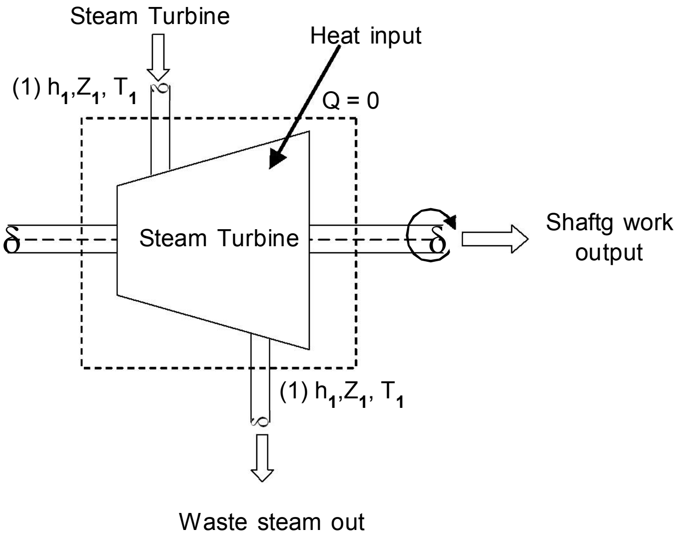
$$h1 + \frac{1}{2}(V_1)^2 = h2 + \frac{1}{2}(V_2)^2$$

If $V_2 >> V_1$, then

$$V_2 = \sqrt{2(h_1 - h_2)}$$

$$= \sqrt{2C_P(T_1 - T_2)}$$

Turbine



A steam turbine receives a superheated, high pressure steam that experiences its. Enthalpy drop as the steam passes over the turbine blades. This enthalpy drop is converted into the kinetic energy of rotation of the blades mounted on the turbine drum. The turbine is well insulated which gives rise to the maximum work output. The turbine is well insulated.

$Q = 0$

Steam velocity at the turbine input = the steam velocity at the output

i.e. $V_1 = V_2$

The turbine is positioned horizontally

$Z_1 = Z_2$

Applying SFEE to the control volume

$H_1 = h_2 + W$

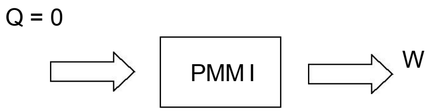
$W = h_1 - h_2$

$= C_P (T_1 - T_2)$

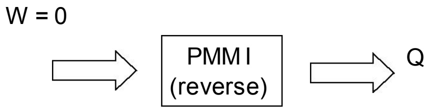
$W = MC_P (T_1 - T_2)$

3.7 perpetual motion machine

PMMI refers to the perpetual motion machine of the first kind. It is a hypothetical machine that will continuously churn out work but without absorbing heat from its surroundings.



But such a machine is not feasible from a practical point of view, for it violates law of conservation of energy (first law of thermodynamics).



The reverse of perpetual machine is also not true. It s a hypothetical machine which is not feasible as if violates the first law of thermodynamics.

Chapter 4.0

SECOND LAW OF THERMODYNAMICS

4.1 Limitation of first law

There are two basic limitations of the first law of thermodynamics

(1) First law does not differentiate between heat and work.

It assumes complete inter-convertibility of the two. Though work being a high grade energy can be fully converted into heat but heat cannot be completely converted to work.

(2) It does not permit us to know the direction of energy transfer. We cannot ascertain whether heat will flow from a higher temperature body to a lower temperature body vice versa.

4.2 Thermal Reservoir

A thermal reservoir is a heat source or heat sink that remains at a constant temperature, regardless of energy interaction.

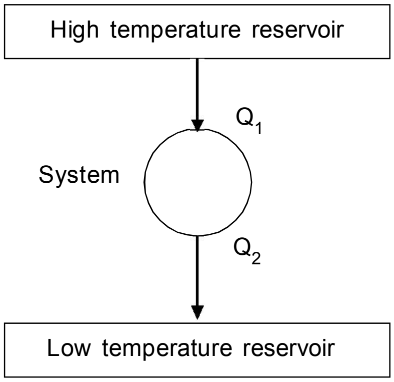
Otherwise a thermal energy reservoir (TER) is a large system body of infinite heat capacity which is capable of absorbing or rejecting a finite amount of heat without any changes in its thermodynamic co-ordinates.

The high temperature reservoir (T_H) that supplies heat is a source.

Sink – Low temperature reservoir to which heat is rejected.

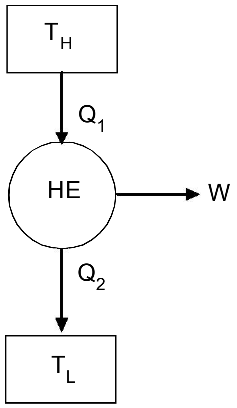
Example

Ocean water and atmospheric air are two good examples.



4.3 Concept of heat engine

A heat engine is a device that can operate continuously to produce work receiving heat from a high temperature T_H and rejecting non-converted heat to a low temperature sink.



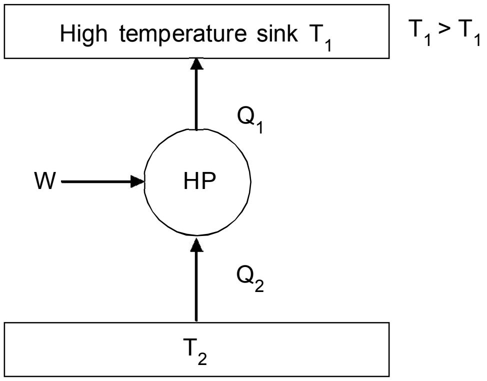
$$\text{Effect} = \frac{\text{output}}{\text{input}} = \frac{W}{Q_1}$$

But, $W = Q_1 - Q_2$ in a cycle

$$\text{So } \eta_{\text{thermal}} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

Heat Pump

A heat pump is a reversed heat engine. It receives heat from a low temperature reservoir (source) and rejects it to high temperature reservoir (since) for which an external work which is supplied to the pump.



The efficiency of a heat pump cycle is usually called the coefficient of performance. It is the desired effect upon the external work supplied for obtaining that desired effect.

$$COP = \frac{\text{Desired effect}}{\text{Work input}}$$

$$COP_{HP} -= \frac{Q_1}{W}$$

$$\text{Again } \sum_{\text{cycle}} Q = \sum_{\text{cycle}} W$$

$$\therefore Q_1 - Q_2 = W$$

$$COP_{HP} = \frac{Q_1}{Q_1 - Q_2}$$

Refrigerator

A refrigerator is similar to a heat pump. It operates as a reversed heat engine. Its duty is to extract heat as much as possible from the cold body and deliver the same to high temperature body.

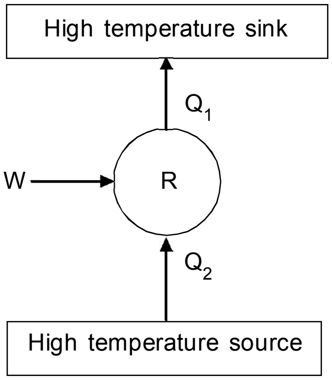
The desired effect of a refrigerator is to remove Q_2 heat infiltrating into the cold space. By using the external work it rejects Q_1 heat to the high temperature reservoir. Therefore,

$$COP_{ref} = \frac{Q_2}{W}$$

$$\text{Again } \sum_{\text{cycle}} Q = \sum_{\text{cycle}} W$$

$$\therefore Q_1 - Q_2 = W$$

$$COP_{ref} = \frac{Q_2}{Q_1 - Q_2}$$



4.4 Statement of second law of the thermodynamics

Clausius statement

It is impossible to construct a device that will produce no effect other than the transfer of heat from a low temperature body to a high temperature body while operating in a cycle.

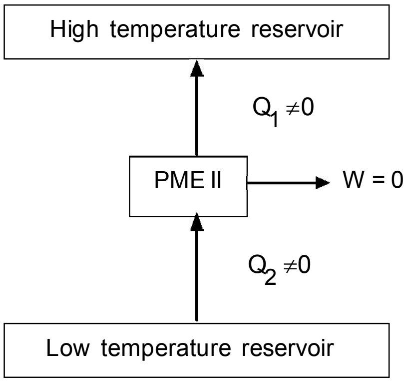
Kelvin Planck statement

No heat engine, operating in cycle, can convert entire heat into work. It is impossible to build a heat engine that can register 100% efficiency.

Note – T_C K-P statement is of relevance to a heat engine. The C-statement relates more directly to a reversed heat engine.

Perpetual motion machine II

It is a hypothetical machine that will continuously pump out heat from a low temperature reservoir (T_2) and delivers the same to a high temperature reservoir at (T_1) without taking up any input work from surroundings.



4.5 Carnot cycle

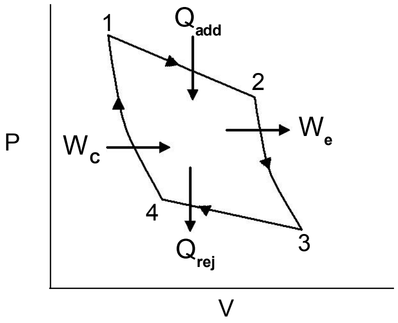
The cannot cycle is a hypothetical cycle developed Nicholas Sadi Carnot (1796-7832) a French military engineer. It is meant for a heat engine or a reversed heat engine. All the process involved in this cycle are reversible, thereby ensuring the best possible device that once could construct. This cycle comprises 4 reversible processes.

Process 1-2 reversible isothermal heat addition

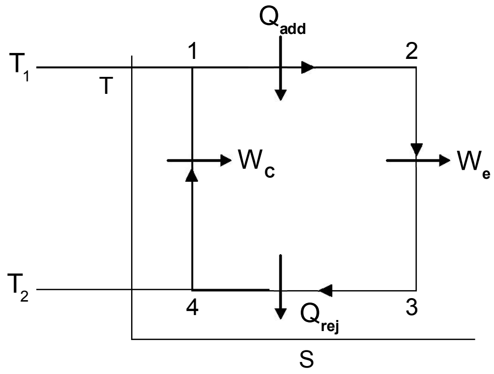
Heat (Q_{add}) flows from a high temperature reservoir to the working fluid which is at a constant temperature but only infinitesimally below that of the source.

$$Q_{add} = \Delta U + W_{1-2},\ \Delta U = 0$$

$$Q_{add} = W_{1-2} \text{ (as isothermal process)}$$



(a) PV diagram



(b) TS diagram

Process 2-3 (adiabatic expansion)

The working fluid expands through a turbine or expander adiabatically producing a net positive work output.

here $Q = 0$

$$\therefore O = \Delta U_{2-3} + W_{2-3}$$

Process 3-4 (Isothermal heat rejected)

Heat (Qrej) is rejected by the fluid to the sink-both one at a constant temperature, but differ only by an infinitesimal amount.

$\Delta U = 0$, isothermal proces

$\therefore -Q_{rej} = -W_{3-4}$

$-Q$ implies heat has been rejected by the system.

$-W$ implies work has been done on the system.

Process 4-1 (Reversible adiabatic compression)

The temperature of the working fluid is raised back to the temperature level of high temperature through adiabatic compression, i.e. Q = 0

$O = \Delta U_{4-1} + \ -W_{4-1}$

$\therefore \Delta U_{4-1} = W_{4-1}$

As the two isothermal and two adiabatic complete the cycle.

So $\sum Q_{net} = \sum W_{net}$

cycle cycle

Or, $Q_{add} + (-Q_{rej}) = W_{1-2} + W_{2-3} - (W_{3-4} + W_{4-1})$

$Q_{add} - Q_{rej} = W_e - W_c$

So, the efficiency

$$\eta = \frac{\text{Net work output}}{\text{Net heat input}} = \frac{W_e - W_c}{Q_{add}}$$

$$= \frac{Q_{add} - Q_{rej}}{Q_{add}} = 1 - \frac{Q_{rej}}{Q_{add}}$$

__Aliter__

Refer to the TS diagram

Ne work done, W_{net} = area 1-2-3-4

$= \text{Side } 1-4 \ \text{ x Side } 1-2$

$= T_1 - T_2 \text{ x } S_2 - S_1$

$Q_{add} = T_1(S_2 - S_1)$

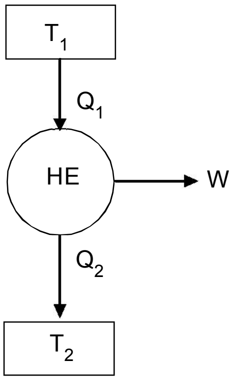
So efficiency, $\eta = \frac{W_{net}}{Q_{add}} = \frac{(T_1 - T_2)\Delta S}{T_1 \Delta S} = 1 - \frac{T_2}{T_1}$

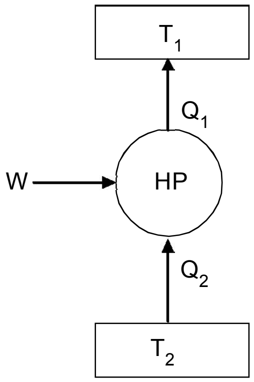
4.6. Application of second law is heat engine, heat pump, refrigerator and determination of cop and efficiencies.

Clausius statement

According to second law without work input heat cannot flow from low temperature to high temperature.

Case of heat engine





Kelvin plank statement

No heat engine operating in a cycle can convert entire heat into work.

in the above engine.

$Q_1 = Q_2 + W$

that means, Q_2 is rejected along with W output

$\Rightarrow Q_1 \neq W$

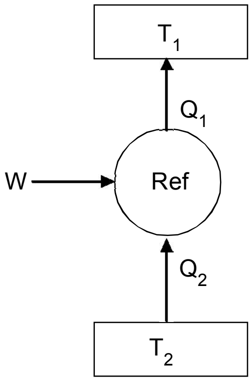
In a heat pump

$W + Q_2 = Q_1$

$W = 0,\ Q_2 = Q_1$

which is impossible, $\Rightarrow W \neq 0$

In a refrigerator



as like heat pump.

$$(COP)_{HP} = \frac{D.E}{Work_{input}} = \frac{Q_1}{Q_1 - Q_2} = \frac{T_1}{T_1 - T_2}$$

as $\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$ (according to carnot's theory)

$$(COP)_{ref} = \frac{D.E}{Work_{ref}} = \frac{Q_2}{Q_1 - Q_2} = \frac{T_2}{T_1 - T_2}$$

$$1 + COP_{ref} = 1 + \frac{T_2}{T_1 - T_2} = \frac{T_1 - T_2 + T_2}{T_1 - T_2}$$

$$\boxed{1 + (COP)_{ref} = COP_{HP}}$$

If installed backward, your household air conditioner will function as a heat pump cooling the surrounding but heating the room.

CHAPTER 5.0

THERMAL ENGINEERING

WORKING SUBSTANCES

Pure Substance

Substance whose chemical composition is uniform throughout its mass.

It should have the following properties.

(i) Homogeneous in composition
(ii) Homogenous in chemical aggregation
(iii) Invariable in Chemical aggregation

(i) Homogeneous in composition

Composition of each part of the system is same.

(ii) Homogenous in chemical aggregation

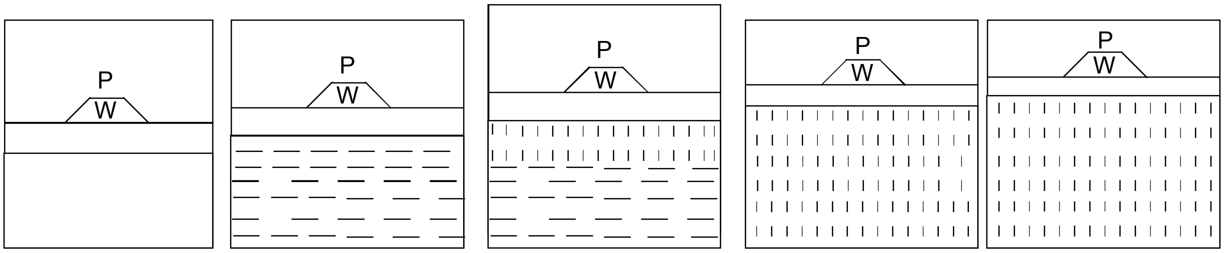
Chemical elements must be combined chemically in the same way in all parts of the system.

(iii) Invariable in Chemical aggregation

The state of chemical combination of the system doesn't change with time.

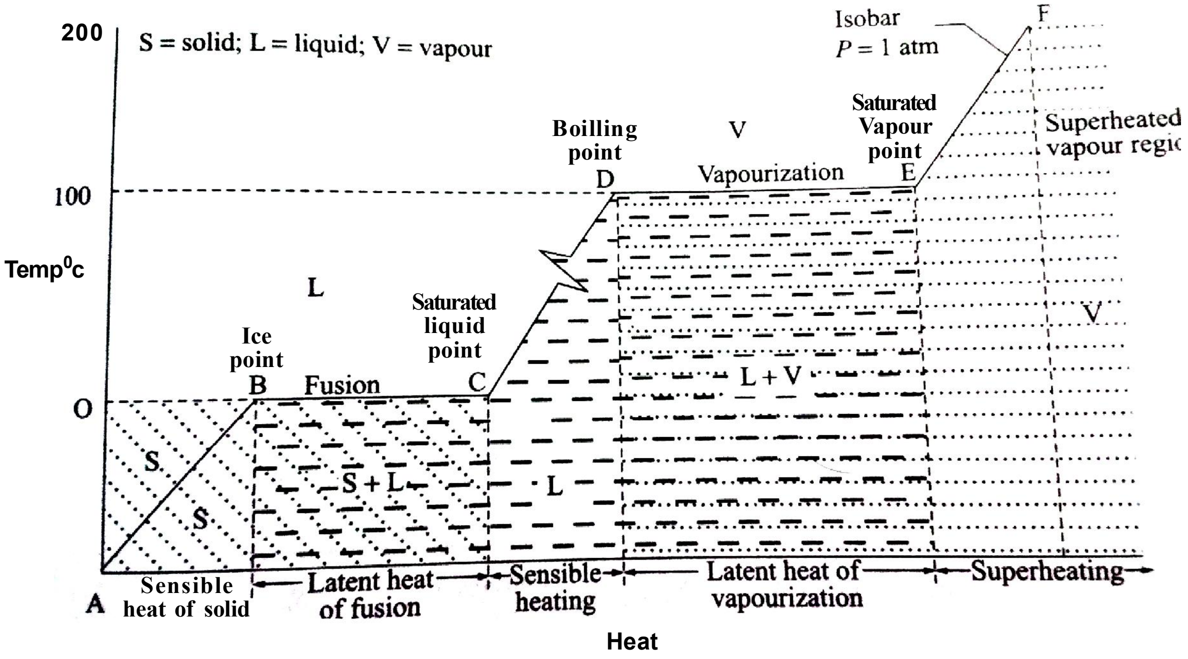
Phase change of pure substance

Let the cylinder & piston machine contains a unit mass of ice at – 10^0c under a pressure 'P' exerted by a weight W placed on the free frictionless piston.



(i) Solid only (ii) Liquid only (iii) Saturated liquid & vapur (iv) Saturated vapur only (v) Super heated vapour

Let the ice be heated slowly so that its temp is always uniform. The changes which occur in the mass of water is as follows :



Process A-B (Sensible heating of ice)

On heating the temp of ice increases from -10^0c to 100^0c. The volume of ice also increases. The point ‘B’ is called as ice point or freezing point of water (0^0c)

Process B-C (Fusion)

On heating the ice melts into water at a constant temp 0^0c. The heat added during the process is called as latent heat of fusion or latent heat of ice.

The volume of the water decreases in comparison with ice 0^0c. This is a peculiar characteristic of water. But all other pure substances increase their volume during this process.

The process in opposite direction (C-B) is called solidification.

Process C-D (Sensible heating of water)

On heating the temp of water increase from 0^0c to 100^0c. The volume of water increases due to thermal expansion.

Process D-E (Vapourisation)

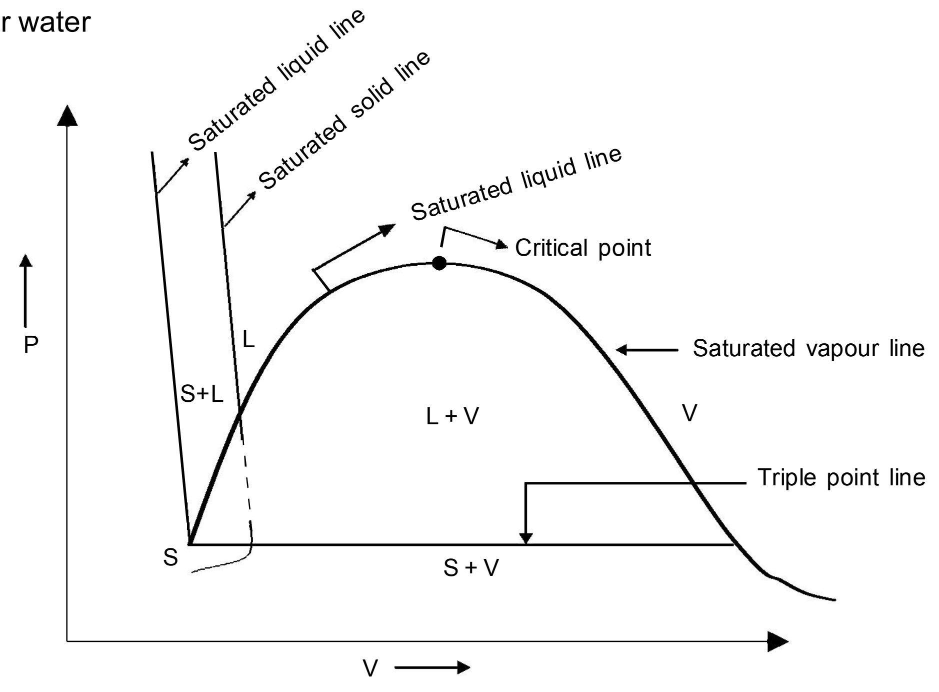
On heating the water starts boiling to vapour at constant temperature of 100^0c. The heat added during the process is called latent heat of vapourisation. The volume of the mixture of water & vapour increases. The vapour at state 5 is dry saturated. The cprocess in opposite direction (E-D) is called condensation.

Process E-F (Superheating)

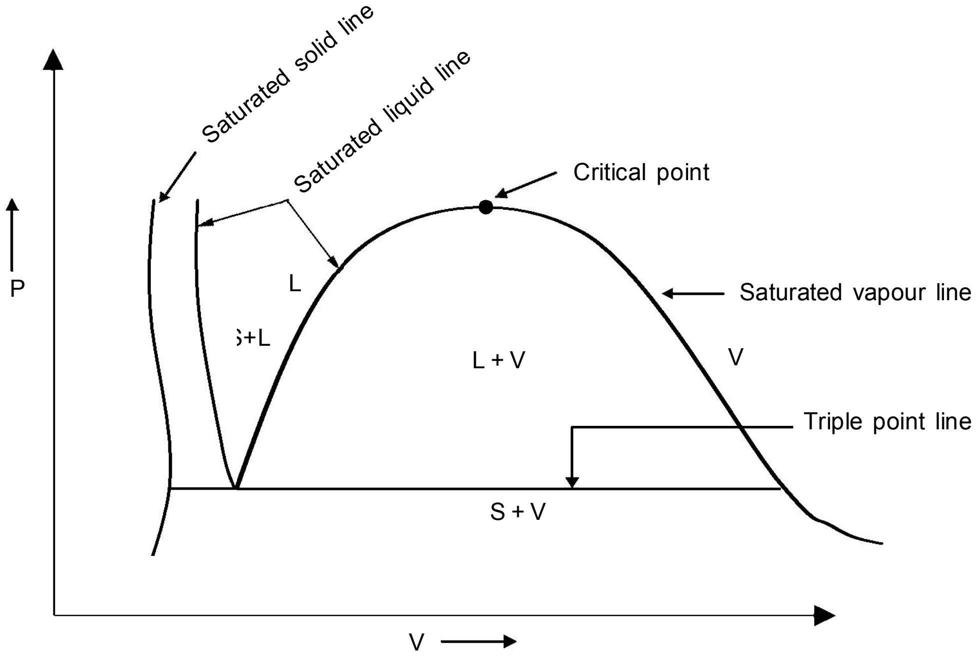
On heating the saturated vapour at E gets superheated & temp of the vapour increases to say 200^0c. The volume of the vapour also increases to height extent. The rise in temperature during the process is called as degree of superheat. The heat added during this process is called as heat of superheat.

P-V diagram

(i) For water



(ii) For all pure substances



Solid-liquid mixture region (S+L)

The space between saturated solid line & saturated liquid line with respect to the solidification is called solid-liquid mixture region.

Solid region (S)

The space left to the saturated solid line is called solid region.

Liquid region (L)

The space between 2 saturated liquid line is called liquid region.

Liquid-vapour mixture region (L+V)

The space between saturated liquid line w.r. t vapourisation & saturated vapour line is called liquid-vapour mixture region.

Triple point line

It is a line on P-v diagram, where all there phases exist in equilibrium.

Solid vapour mixture region (S+V)

The space below the triple point line is called solid vapour mixture region.

Critical point

The point at which the saturated liquid line & saturated vapour line meet each other is known as critical point.

Above the critical point, the liquid on heating suddenly converted into vapour or the vapour on colling suddenly converted into liquid.

The pressure, temperature & volume w.r.t critical point are known as critical pressure, critical temp & critical volume respectively.

For water

$P_C = 221.2$ bar $T_C = 374.150c$, $V_C=0.00317\ m^3 / kg$

Saturated temperature

The temperature at which a pure liquid (at a given pressure) changes into vapour.

Saturated pressure

At a give temperature a pure liquid changes into pressure. That particular pressure is called saturated pressure.

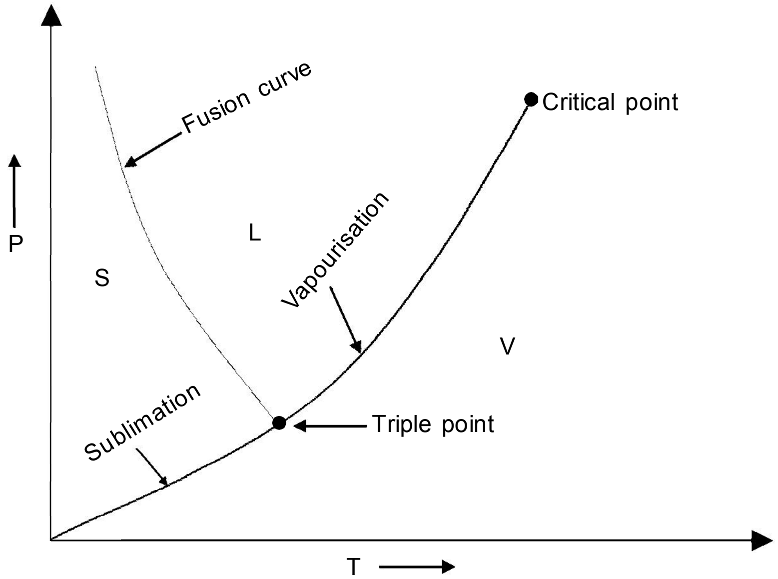
P-T diagram

(i) For water

T_t = 273.16K

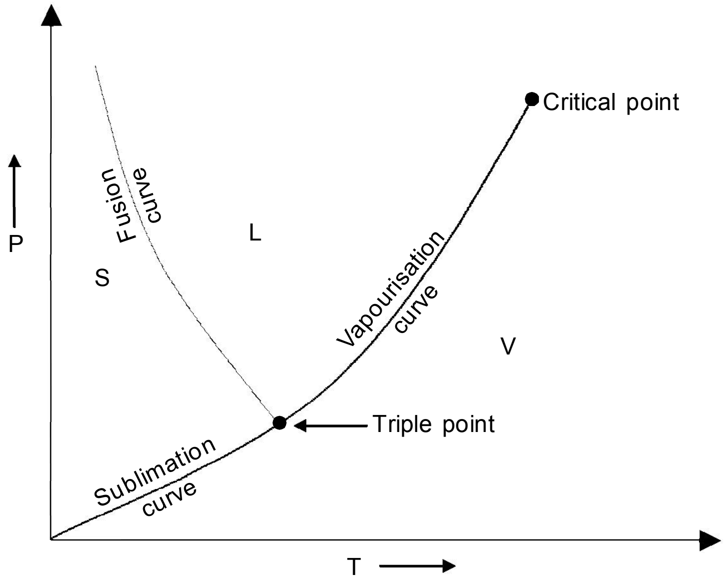
P_t = 4.58mmHg

(t = triple point)



T_t = 273.16 K, P_t = 4.58mmHg, (t = triple point)

(ii) For pure substances



Specified heat :

(i) It is defined as the amount of heat required to raise the temperature of a unit mass of substance through unit degree.

(ii) $C = \frac{\theta}{M \times \Delta t} \quad \text{unit} (J/kg-k)$

M = mass of substance

Δt = rise or fall of temperature.

(iii) Specific heat at constant pressure is called C_p. Specific heat at constant volume is called Cv.

(iv) It doesn't depend upon path.

Quality or dryness fraction :

It is defined as the ratio of mass of dry steam actually present to the total mass of the vapour which contains it.

$$\text{Dryness fraction} = \frac{\text{dry steam mass}}{\text{total mass of vapour}}$$

$$X = \frac{M_g}{M_g + M_f} = \frac{M_g}{M}$$

Mg = mass of dry vapour, Mf= mass of liquid particle, M = total mass = (M_g+M_f)

$(1-\chi)$ = wetness fraction

$$\text{wetness fraction} = \frac{\text{mass of liquid particle present}}{\text{total mass of steam}}$$

For dry saturated vapour, quality = 100% i.e. $\chi=1$

(i) $V = V_f + V_g$

V_f = volume of liquid particle present

V_g = volume of vapour present.

$\boxed{V = V_f + \chi V_{fg}}$ $V_{fg} = V_g - V_f$

$\boxed{S = S_f + \chi S_{fg}}$ $S_{fg} = S_g - S_f$

$\boxed{u = u_f + \chi u_{fg}}$ $h_{fg} = u_g - h_f$

Heat of superheat $(h_s) = C_p (T_{sup} - T_{sat})$

$T_{sup} - T_{sat}$ = degree of superheat

Steam Table

The properties of water are arranged in the steam table as the function of temperature & pressure. If given temp ⟩ steam table temperature.

Then it is a case of superheat.

$$V = V_1 + \frac{V_2 - V_1}{T_2 - T_1} x \text{ given } T$$

$$h = h_1 + \frac{h_2 - h_1}{T_2 - T_1} x \text{ given } T$$

$$S = S_1 + \frac{S_2 - S_1}{T_2 - T_1} x \text{ given } T$$

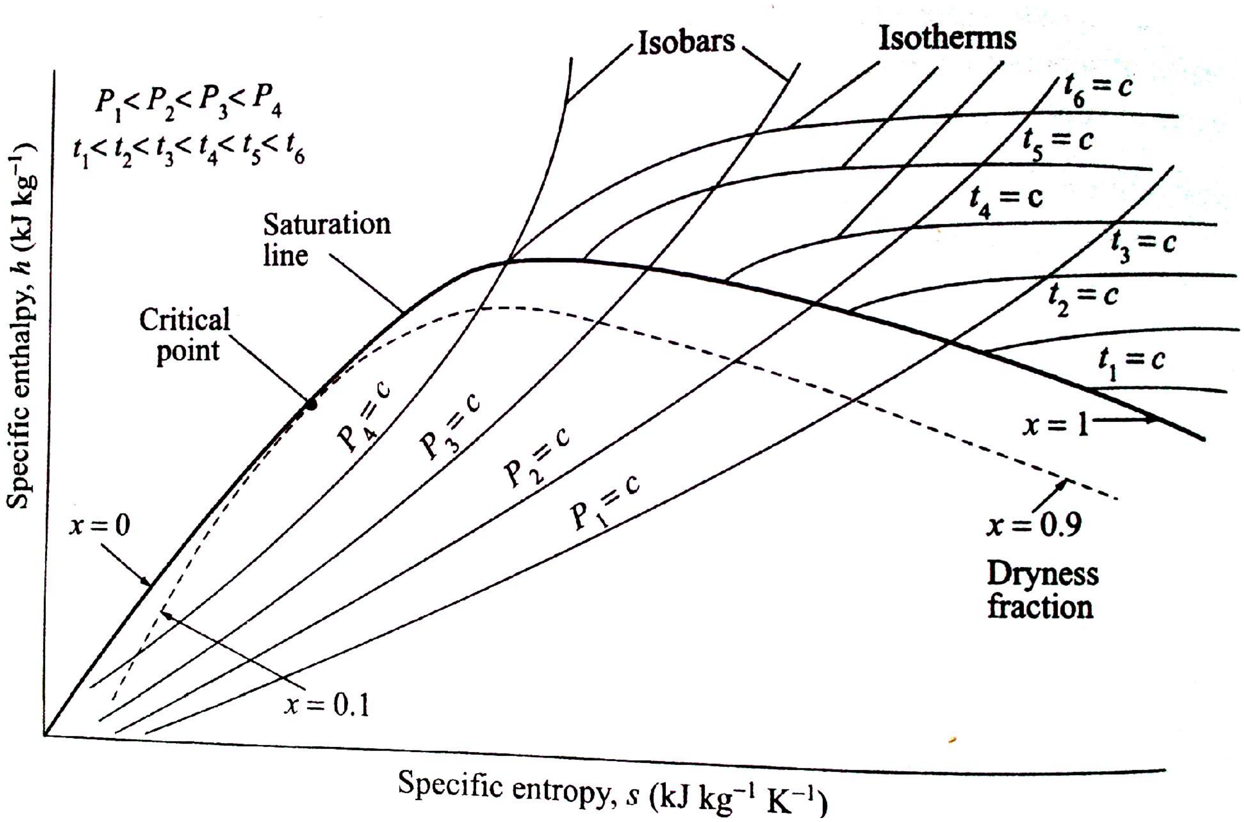
Similarly if given volume > steam table volume. It is a case of superheat

$$T = T_1 + \frac{T_2 - T_1}{V_2 - V_1} x \text{ given } V$$

$$h = h_1 + \frac{h_2 - h_1}{V_2 - V_1} x \text{ given } V$$

$$S = S_1 + \frac{S_2 - S_1}{V_2 - V_1} x \text{ given } V$$

Mollier diagram



Constant pressure line = P_1, P_2, … etc.

Constant temperature line = t_1, t_2.. etc.

CHAPTER 6.0

IDEAL GASES & REAL GASES

6.1. Boyle's Law

The volume of a given mass of gas is inversely proportional to its absolutely pressure at constant temperature.

i.e. $V \alpha \frac{1}{P} \; (T = \text{constant})$

$$\boxed{PV = \text{constant}}$$

Charles's Law

The volume of a given mass of a gas directly proportional to its absolute temp at constant pressure.

$\text{i.e. } V \alpha T \;\; (P = \text{constant})$

$$\boxed{\frac{V}{T} = \text{Constant}}$$

Ideal gas Law

From boyle's law PV=C

Charle's law $\frac{V}{T} = C$

Combining both the law

$$\frac{PV}{T} = C$$

$\text{i.e. } \boxed{PV \alpha T}$

$$\boxed{PV = RT} \quad .....(i)$$

This equation is called characteristic gas equation or ideal gas equation.

R= characteristic gas consant

= 0.287 KJ / Kg-k (for atm. air)

Universal gas constant (Ru).

In general PV= mRT (m=mass of gas)

But m = nM

n = no of kg moles

M = Molecular mass of the gas

.. PV = nMRT

$= n(MR)T = nR_uT....$

R_u = universal gas constant = M.R

M.R = constant of all the gases.

$$\boxed{R_u = 8.3143 \text{ KJ/ Kgk}}$$

Avogadro's law :

It states that the equal volumes of different ideal gases at the same temperature & pressure contains equal number of molecules.

$$n=\frac{m_1}{M_1}=\frac{m2}{M2}=\frac{m3}{M_3}\ldots$$

$$\text{but}\, M=\rho v$$

$$n=\frac{\rho 1}{M_1}=\frac{\rho 2}{M_2}=\frac{\rho 3}{M_3}$$

$$\rho=\text{Massdensity}=\frac{1}{V}$$

$$V=\text{Specific volume}$$

$$n=\frac{1}{M_1 V_1}=\frac{1}{M_2 V_2}=\frac{1}{M_3 V_3}\ldots$$

$$M_1V_1=M_2V_2=M_3V_3=\text{Cons tan t}$$

SP volume molecular weight = Molar volume

$$\boxed{V.M=\overline{V}}$$

Dalton's law :

(i) The pressure of a mixture of gasses is equal to the sum of the partial pressure of the constituents.

(ii) The partial pressure of each constituent is that pressure which the gas would exact if it occupied alone at that given volume occupied by the mixture at the same temperature.

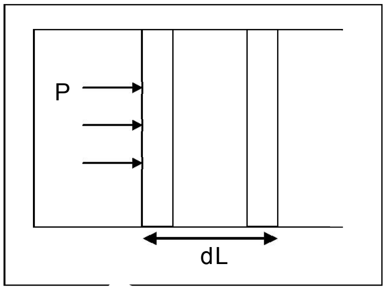
$$\boxed{P= P_A + P_B}$$

(iii) It is found the dalton's law is more accurately by gas mixtures at low pressure.

$$P = P_A + P_B + \ldots\ldots\ldots P_n = \Sigma P_i$$

P_i = Partial pressure of the constituent.

Work done in moving the boundaries of a closed system



Consider a gas of contained in a piston-cylinder arrangement pressure exerted by the gas is P on the cross sectional area of piston A.

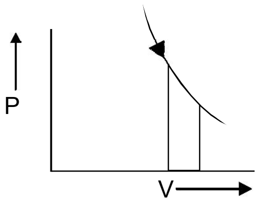
Let the piston moves a distance dl

Work done = Force x displacement

$= (P.A) \times dL \qquad \left(P=\frac{F}{A}\right)$

$= P.\, dv \qquad (A.dL=dv)$

Suppose, gas expands from 1 to 2

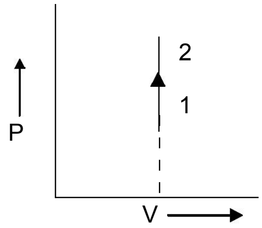


$$\text{the W.D} = \int_{V_1}^{V_2} P.dv$$

W.D by the system is +ve
W.D on the system is –ve

Constant volume process

W.D = P. dv = 0
dw = 0



$$dQ = MC_V = dT \text{ (heat supplied)}$$

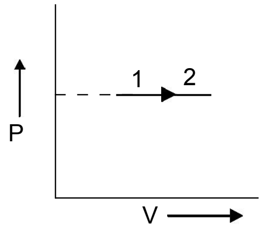
According to 1st law of thermodynamics

$$dQ - dw = du$$

$$MC_v.dT - 0 = du$$

$$\boxed{\Delta U = MC_v (T_2 - T_1) \text{ (Isochoric process)}}$$

Constant pressure process (Isobaric process)



$$dw = \int_1^2 Pdv$$

$$= P(V_2 - V_1)$$

$$= P_2V_2 - P_1V_1 = MR(T_2 - T_1)$$

$$\because P_2 = P_1 = P$$

$$dQ = \int_1^2 mC_P dT$$

$$= mC_P (T_2 - T_1)$$

$$dU = mC_P (T_2 - T_1) - mR(T_2 - T_1)$$

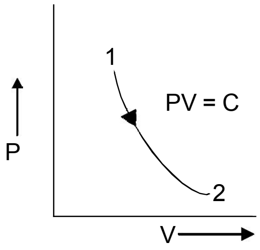
$$= mC_V (T_2 - T_1)$$

$$\because C_P - C_V = R$$

Constant tempretaure process (Isothermal process)

$$PV = C$$

$$P = C/V$$



$$dw = \int_{V_1}^{V_2} Pdv$$

$$= \int_{V_1}^{V_2} \frac{C}{V} dv$$

$$= C[\text{Inv}]\frac{V_2}{V_1}$$

$$= C(\text{Inv}_2 - \text{Inv}_1)$$

$$= C\text{In}\left(\frac{V_2}{V_1}\right) = P_1V_1\text{In}\left(\frac{V_2}{V_1}\right) = P_2V_2\text{In}\left(\frac{V_2}{V_1}\right)$$

$$= mRT\text{In}\left(\frac{V_2}{V_1}\right)$$

$$dU = mC_V (T_2 - T_1) = 0$$

$$\because T_2 = T_1$$

$$Q - W = \Delta U = U_2 - U_1$$

$$\boxed{Q = W}$$

Isentropic process

No heat transfer between the system & the surrounding.

Constant pressure process (Isobaric process)

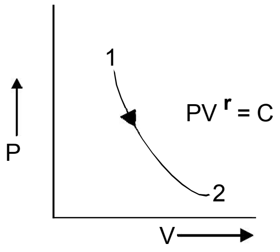
$$PV^{\gamma} = C$$

$$dw = \int_1^2 P.dv = C\int_{V_1}^{V_2} \frac{dv}{V^{\gamma}}$$

$$= C\left(\frac{V^{-\gamma+1}}{V^{-\gamma+1}}\right)_{V_1}^{V_2}$$

$$= \frac{C}{1-\gamma}\left(P_2V_2^{\gamma}.V_2^{1-\gamma} - P_1V_1^{\gamma}.V_1^{1-\gamma}\right)$$

$$= \frac{P_1V_1 - P_2V_2}{\gamma-1}\left(C = P_1V_2^{\gamma} = P_2V_2^{\gamma}\right)$$



$$Q = 0$$

$$\boxed{\Delta U = W}$$

Vender walls equation

$$\left(P + \frac{a}{\nu^2}\right)(\nu - b) = RT$$

a,b = Specific constant & depend upon the type of the fluid.

V = volume per unit mass, R = gas constant.

If volume of 1 mole is considered

$$\left(P + \frac{a}{\nu^2}\right)(\bar{\nu} - b) = R_0T$$

$P, \bar{V}, T, R, a, b$ units are as follows

$$P = N/M^2, \bar{V} = M^3/kg-k, T = K, R = 8314NM/kgmol-k$$

$$a, b = NM^4/(kgmol)^2, b = M^3/kg$$

Difference between Real gas & Ideal gas

Real gas	Ideal gas
(i) No gas flows all conditions of gas law under certain conditions of pressure & temperature but it approaches the ideal gas behavior. Hence it is called real gas.	(i) A gas which obeys all the laws of gas under all conditions of temperature & pressure.
(ii) The relations derived from ideal gas may be applied to real gases under certain condition.	(ii) Ideal gas law is simple & hence simple relations are derived from it.
(iii) In certain condition real gas no more remain in gaseous phase but changes its state. So real gases under certain condition behaves like ideal gas.	(iii) O2, N2, H2 may be treated as ideal gas because ordinarily these are difficult to liquefy.

Thermodynamics:-

ESE:

Paper I
- Objective
  18 Questions
  30 marks
- Conventional
  1 Question
  40 marks

70 marks

GATE
- 15 Math
- 15 Aptitude
- 70 ME
  ↓
  last yr. 8 marks

Basic Concepts
Zeroeth law of thermodynamics
first law of thermodynamics
first law applied to flow process
2nd law of thermodynamics.
Entropy
Availability of Energy
Properties of Pure substances

Text Books,
Thermodynamics – P.K. Nag.
(Solve even unsolved questions).

④

-: BASIC CONCEPTS :-

Thermodynamics :- It is the science of energy transfer and its effects on properties of system.
The main aim of thermodynamics study is to convert disorganised form of energy (heat) into organised form of energy (work) in an efficient manner.

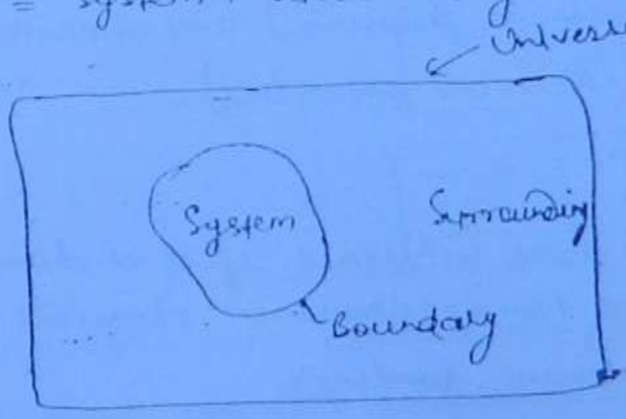
System :- It is a region in space upon which the study is focussed or concentrated.

Surroundings :- Anything external to the system is known as surroundings.

Boundary :- The separation between systems and surrounding is known as boundary.
Boundary can be either "rigid or movable" and boundary can be "real or imaginary."

Note : Universe = System + Surrounding

⑤



** TYPES OF SYSTEM :

Types of System	Mass Transfer	Energy transf	Examples
Closed	✗	✓	Piston cylinder without valves.
Open	✓	✓	Turbine, Comp, Pumps, etc.
Isolated	✗	✗	Universe, Hot coffee in a well insulated flask

* Microscopic and Macroscopic approach of Thermodynamics :-

In microscopic approach, the behaviour of individual molecules is taken into consideration. This approach is also known as statistical thermodynamics. and this approach is useful at low densities (higher altitudes).

$$\boxed{\rho = \frac{\overset{\rightarrow \text{No. of molecules}}{m}}{v}}$$

In macroscopic approach, the behaviour of individual molecules is not taken into consideration but the average behaviour of molecules is taken into consideration. This approach is also known as classical thermodynamics.

In Our course we follow Classical Thermodynamics. ⑥

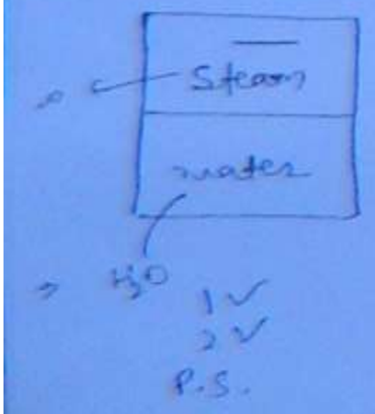
* Thermodynamic Equilibrium :-

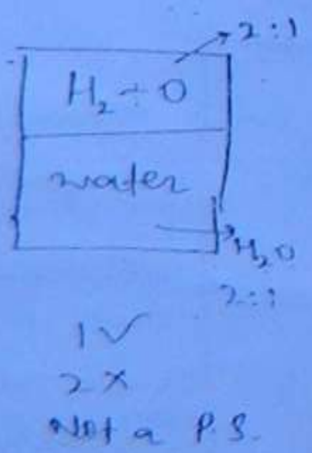
A system is said to be in thermodynamic equilibrium, if it is in thermal equilibrium (equality of temperature), mechanical equilibrium (equality of force or pressure) and chemical equilibrium (equality of chemical potential). -

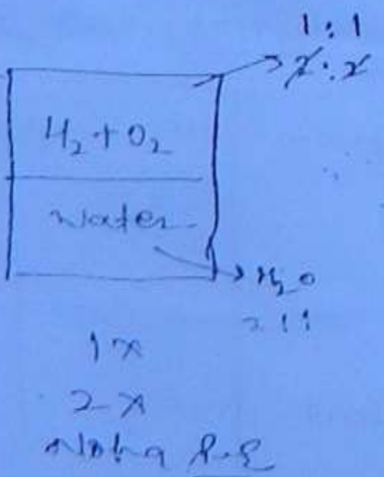
* Pure Substances :-

A substance is said to be pure substance if it is homogeneous in chemical composition and homogeneous in chemical aggregation.

* (Aggregation means bonding).







⇒* PROPERTIES OF A SYSTEM :- (Important).

Properties are characteristics of system. Examples Pressure, tempr., volum density, mass, etc. Properties are of two types:-

① Intensive (or intrinsic)
② Extensive (or extrinsic)

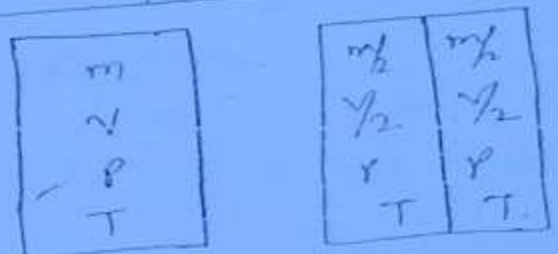
Intensive properties are independent of size or mass.

Examples - pressure, temperature, density, thermal - - conductivity, viscosity, etc.

Extensive properties depend on size or mass of the system.

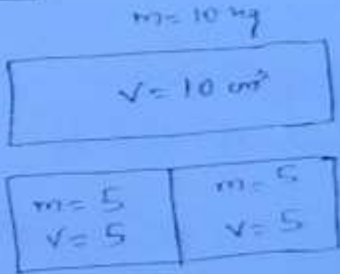
Examples - volumes, all forms of energy, etc.

Note :- Ratio of two extensive properties is an intensive property.



⑦

$$\overset{I}{\uparrow}\ \rho = \frac{m \rightarrow E}{V \rightarrow E}$$



$\rho = m/V = 1.$

$\rho = m/V = 1.$

Note :- Specific properties are extensive properties per unit mass and these ~~specific~~ properties are intensive propertie

Specific Properties are intensive properties.

Example :- specific volume (v)
specific enthalpy (h)
specific internal energy (u)
specific entropy (s).

* **Important points with respect to properties :-**

① Properties are point functions or state functions.
② Properties are independent of past history.
③ Properties are exact differentials.

⑧

3* **State of a system :-** The condition of a system is known as state of a system. The condition of a system is specified by means of its properties. As long as the properties are fixed, the condition is also fixed.

Process :- A change of state is known as a process.

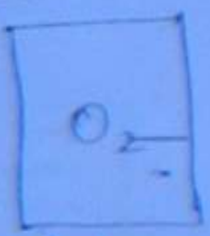
* **GIBB'S PHASE RULE :-**

According to Gibb's phase rule :-

$$\boxed{P+F=C+2}$$

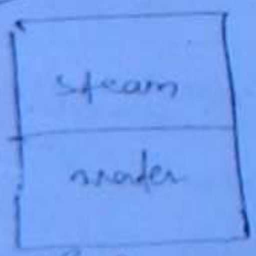
where,
P = No. of Phases
F = Degree of freedom
or
Minimum no. of independent intensive variables required.
C = No. of components.

Ex.1



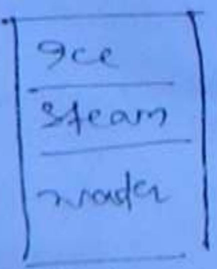
$P=1$
$C=1$
$P+F=C+2$
$1+F=1+2$
$\therefore \boxed{F=2}$

Ex.2



$P=2$
$C=1$
$P+F=C+2$
$\boxed{F=1}$

Ex.3



$P=3$
$C=1$
$P+F=C+2$
$\boxed{F=0}$

⇒* THERMODYNAMIC CYCLES :-

A system is said to have undergone a cycle if the initial and final points are same. Minimum number of processes required for a cycle are 2.

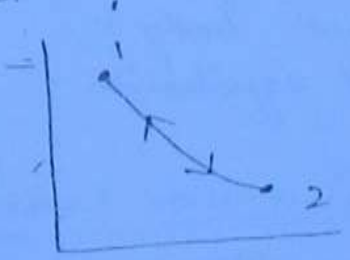
Note:- for a cycle the change in property is equal to zero. becau property depend on point and for a cycle initial and final points are same.

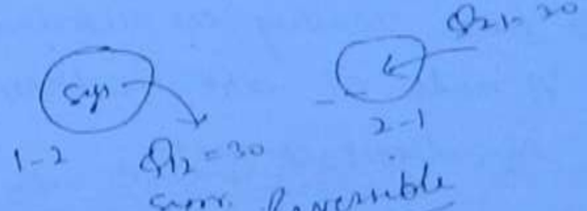
⇒* REVERSIBLE AND IRREVERSIBLE PROCESSES :-

A process is said to be a reversible process, if when reversed in direction follows the same path as that of the forward path without leaving any effect on system and surroundings.

A process which is not a reversible process is an irreversible process.

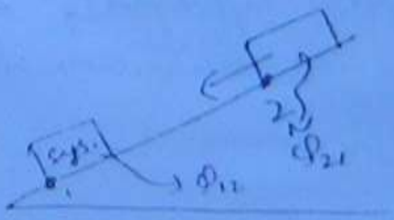
Friction is one of the reasons which makes a process irreversible. Reversible process is the most efficient process.





Reversible

System	Surroundings
$Q_{12} = -30$	$Q_{12} = +30$
$Q_{21} = +30$	$Q_{21} = -30$
0	0



Irreversible

⇒ * QUASI - STATIC PROCESS :-

$\underset{\text{(almost)}}{\text{QUASI}}$ - $\underset{\text{(rest)}}{\text{STATIC}}$

A process is said to be Quasi-static process, if it is carried out in a very slow manner with small gradient. Frictionless Quasi-static process is a reversible process.

⑩

⇒ * ZEROTH LAW OF THERMODYNAMICS (Concept of Temperature)

Statement :- When a body 'A' is in thermal equilibrium with body 'B' and body 'B' is in thermal equilibrium with body 'C' separately, then 'A' and 'C' are in thermal equilibrium.

In zeroth law of thermodynamics one body acts as thermometer. Let us assume that body 'B' is thermometer. Let us emerge thermometer 'B' in body 'A'. When the thermal equilibrium is reached, body 'B' records some temperature. Now, place the body 'B' (thermometer) in body 'C', If the thermometer records same reading as recorded with body 'A', then we can say that 'A' and 'C' are in thermal equilibrium without bringing them in direct contact.

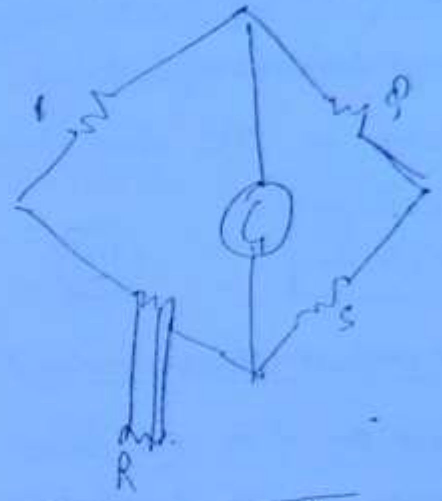
* Principle of Thermometers :-

In finding temperature, first the property which varies with temperature is found and this property is known as thermometric property and with the help of thermometric property, the unknown temp. 'T' can be found.

⇒ * Types of thermometers :-

① Resistance Thermometer (Thermistor) :

These thermometers are based on wheat stone bridge principle. In these thermometers, resistance plays the role of thermometric property.



$P/Q = R/S$

$R = \frac{PS}{Q}$.

11

$$\boxed{R = R_0(1 + \alpha t + \beta t^2)}$$

② <u>Thermocouple</u> :- Thermocouples are based on Seebeck effect. According to this Seebeck Effect, when two dissimilar metals are joined to form two separate junctions and if these junction are maintained at different temp emf (voltage) is generated as this ~~temp~~ emf is proportional to two temperature difference

③ Constant Volume Gas ~~ther~~ thermometer :- In constant volume gas thermometer, pressure plays the role of thermometric property.

④ Constant Pressure Gas thermometer :- Volume plays the role of thermometric property.

NB Type of thermometer — Thermometric Property

1) Thermistor →

2) Thermocouple →

3) constant Vol^m Gas thermometer →

4) Constant Pressure " " →

* Temperature Scale:-

Temperature scales are arbitrary.

In celsius scale, ice point is taken as 0°C and steam point is taken as 100°C.

$$\boxed{K = \_\,^{\circ}C + 273.15}$$

⑫

Method used before 1954 for temperature measurement :-

This method is based on two reference temperatures i.e. ice point (0°C) and steam point (100°C).

$$t = ap + b$$

$$t_i = ap_i + b$$

$$t_s = ap_s + b.$$

$$100 = ap_s + b$$

$$\underline{\;0 = ap_i + b\;}$$

$$100 = a(p_s - p_i)$$

$$\text{or}\quad a = \frac{100}{(p_s - p_i)}$$

Now

$$0 = ap_i + b$$

$$\text{or,}\quad b = -ap_i$$

$$= -\frac{100}{(p_s - p_i)} \times p_i$$

$$\therefore\quad t = \frac{100}{(p_s - p_i)} p - \frac{100}{(p_s - p_i)} . p_i$$

$$\therefore\quad \boxed{t = \frac{100}{(p_s - p_i)} . (p - p_i)}$$

By knowing the unknown property "p", the temperature 't' can be found.

* **Method used after 1954 for temperature measurement :-**

It is based on single fixed temperature i.e. triple point of water.

$$P\overset{c}{V} = \overset{c}{m}\overset{c}{R}T$$

let $V = C$.

$$P \propto T$$

$$P = cT.$$

$$c = P/T$$

At triple point,

$$c = \frac{P_{tp}}{T_{tp}}.$$

$$\therefore P = \frac{P_{tp}}{T_{tp}} \times T.$$

$$T = \frac{P}{P_{tp}} \cdot T_{tp}$$

$$\boxed{T = 273.16\left(\frac{P}{P_{tp}}\right)}$$

At triple point.

$T = 0.01°C$

$K = 0.01 + 273.15$

$K = 273.16\,K.$

The triple point of water is assigned a value of 0.01°C (273.1

Note:- Ideal gas thermometers are independent of material of construction. ∵ All ideal gas behave same.

PROBLEM

1. The reading 't_A' and 't_B' of 2°c thermometer 'A' & 'B' agree at ice point (0°C) and steam point (100°C) and are related by the equation, $t_A = l + m t_B + n t_B^2$. Between these temp where l, m and n are constants when both are immersed in oil 'A' reads 51°C and 'B' reads 50°C. Determine the reading on 'A', when 'B' reads 25°C. And discuss the question which thermometer is correct?

S:
$$t_A = l + m t_B + n t_B^2$$

$$t_A = 0°C = t_B$$

⑭

$$0 = l + m(0) + n(0)$$

$$\Rightarrow \boxed{l = 0}$$

$$t_A = m t_B + n t_B^2$$

$$t_A = 100°C = t_B$$

$$100 = m(100) + n(100)^2$$

$$1 = m + 100n \quad \text{—①}$$

$$t_A = 51°C, \quad t_B = 50°C$$

$$51 = 0 + m(50) + n(50)^2$$

$$\Rightarrow 51 = 50m + 2500n \quad \text{—②}$$

from ① & ②

$$m = 1.04$$

$$n = -4 \times 10^{-4}$$

$$t_A = 0 + 1.04 t_B - 4 \times 10^{-4} t_B^2 \quad \text{—③}$$

$t_B = 25°C$

$$t_A = 1.04(25) - 4 \times 10^{-4}(25)^2$$

$$\boxed{t_A = 25.75°C}$$

Though the end point temperatures are same, it does not necessarily mean the intermediate point temperatures are also same.

① **Assertion (A):-** If alcohol and mercury thermometers read exactly at ice point and steam point, these two thermometers will also give same reading exactly at 50°C.

Reasoning (R):- Temperature scales are arbitrary.

Ans:- Assertion is wrong and Reasoning is independently correct.

② Which of the following are intensive properties?

1. Kinetic Energy 2. Thermal conductivity
3. ~~Volume~~ Pressure 4. Volume.

15

Ans: (b) (2 and 3)

③ Match the following Question:-

List I (Type of thermometer)	List II (Thermometric Property)
A) Mercury in Glass	1) Pressure
B) Thermocouple	2) Resistance
C) Thermistor	3) Volume or length
4) Constant Vol^n Gas Thermometer	4) Emf

Ans:- 3-4-2-1

④ The thermometric property x varies with temp^n T according to the relation $t = ax^2 + b$, where 't' is in °C and 'x' is in cm. ~~a~~ a and b are constants. At ice point (0°C) and steam point (100°C) the values of x are 5cm and 20cm respectively. When this thermometer is brought in contact with heated body, the value of 'x' is recorded as 15 cm. Then the temp^n of heated body is?

Soln:

$0 = 25a + b \Rightarrow b = -25a$ — ①

$100 = 400a + b$

$\Rightarrow 100 = 400a - 25a \Rightarrow a = 100/375 = 0.27$

$\therefore b = -25 \times 0.27 = -6.75$

$$t = 0.27(15)^2 + -6.75$$

$$\Rightarrow \boxed{t = 54^\circ C} \qquad \simeq \boxed{53.3^\circ C} \;\cdots$$

$a = 0.26$
$b = -6.66$

V.V.I
P-5 ** In a new temperature scale, °P, the boiling and freezing points of water are 100°P and 300°P respectively. Find the reading corresponding to 0°P on celcius scale.

Sol^n Let us assume a linear relation between °C and °P scale.

let it be,

$$^\circ P = a \times {}^\circ C + b.$$

⑯

Now

$$300 = a \times (0) + b$$

$$\Rightarrow \boxed{b = 300}$$

Again

$$100 = a \times 100 + 300$$

$$\Rightarrow \boxed{a = -2}$$

$$\therefore {}^\circ P = -2 \times {}^\circ C + 300$$

when °P

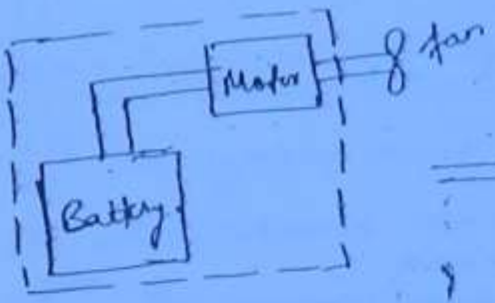
$$0 = -2 \times {}^\circ C + 300.$$

$$\therefore \boxed{{}^\circ C = 150^\circ}$$

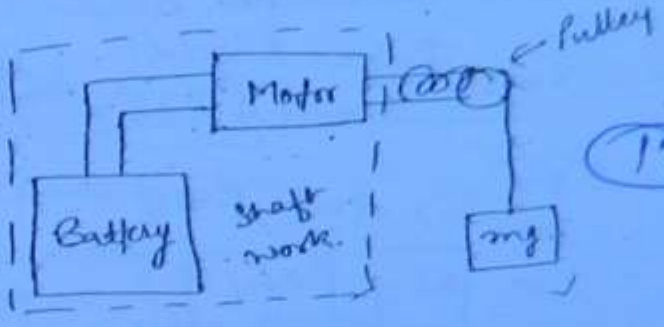
CHAPTER-2 :-

ENERGY INTERACTIONS (WORK & HEAT)

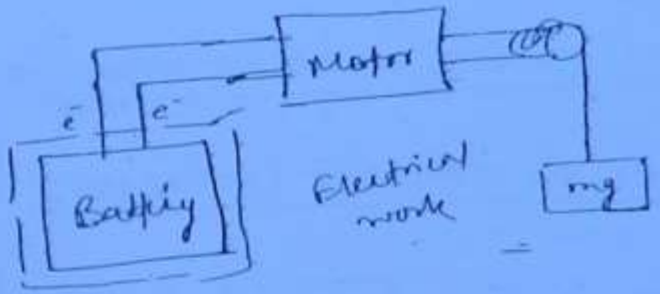
⇒* Thermodynamic work :- Work is said to be done by the system if the sole 'effect on things external to the system can be equated to raising of weights (weights may not be actually raised but the effect can be equated to raising of weights).

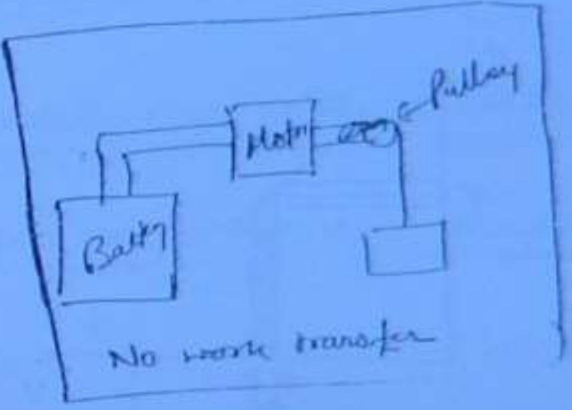


=



17

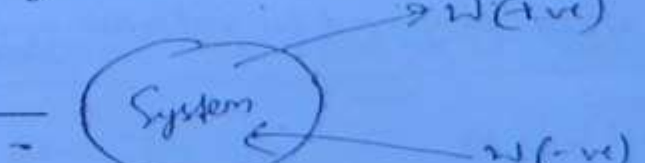




No work transfer

⇒* Conventions of work transfer :-

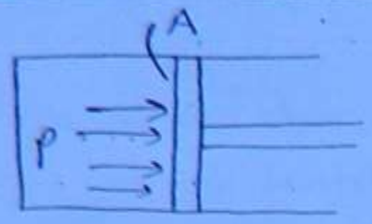
Work done by the system is taken as positive and work done on the system is taken as negative.



+ve work (raising of weights)
(Boundary weights)

-ve work (lowering of Boundary contracts. weights)

⇒ * Generalised Equation for Non-flow work or closed system work:-



$P = F/A$

$F = PA$

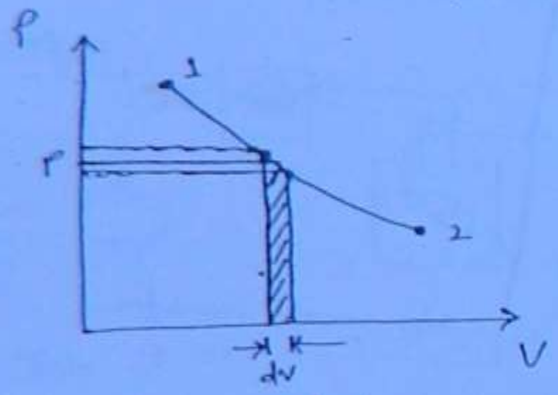
$W = F \times x$

$dW = F \times dx$ ⑱

$= \underbrace{P.A \times dx}$

$\therefore dW = P.dV$

or $\boxed{W = \int P dV}$ → Non-flow work / (Closed system work).



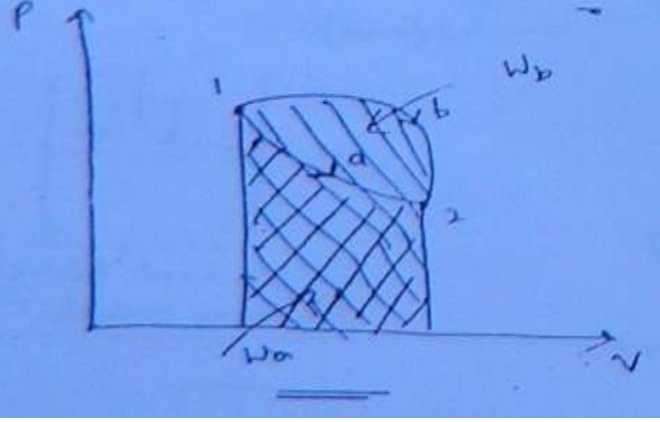
∴ Area = $P.dV$

∴ Also, Work = $P dv$.

∴ $\boxed{\text{Area} = \text{Work}}$

ery Important Sentence)

⇒ Area under the curve when projected on volume axis gives closed system work.



$W_a \neq W_b$

→ Though the end points for 'a' and 'b' are same, work transfer is not same because areas are different. Therefore, work transfer depends on path followed by the process and hence work transfer is a path function and it is not a property and it is inexact differentials ($đW$ or δW).

⇒* There is no work transfer when the system is in equilibrium state. Work transfer is always associated with a process.

⇒* Conditions for Applying the Equation $W = \int P dV$:→

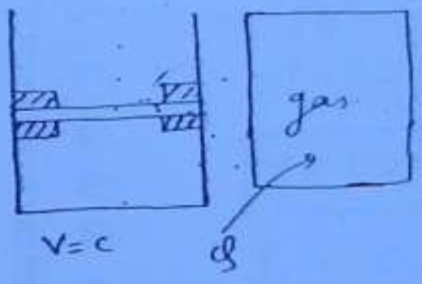
① The system must be a closed system.

② Work should cross the boundary.

③*** The process must be a reversible process.

⇒* NON-FLOW OR CLOSED SYSTEM WORK FOR VARIOUS PROCESSE

① CONSTANT VOLUME OR ISOCHORIC OR ISOMETRIC PROCESS:



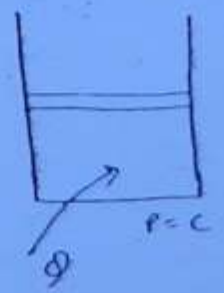
$V = C \Rightarrow dV = 0$

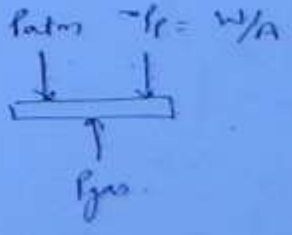
$W = \int P dV$

$W = 0$

Constant volume closed system work = 0.

② CONSTANT PRESSURE OR ISOBARIC OR ISOPIESTIC PROCESS:





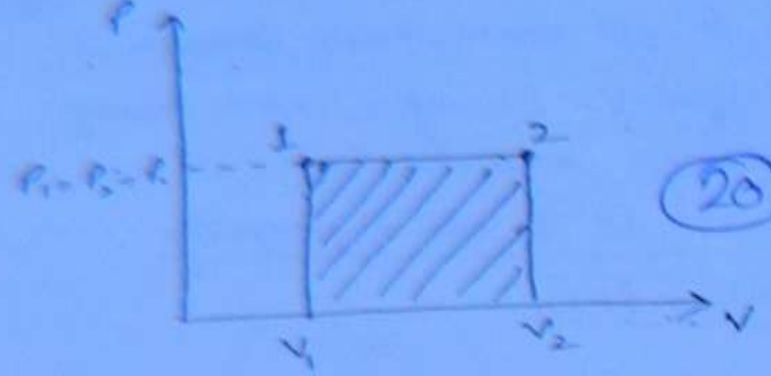
At equilibrium.

$P_{atm} + P_P = P_{gas}$

$P_{atm} + \frac{W}{A} = P_{gas}$

∵ P_{atm}, 'W' & 'A' are constants.

So, P_{gas} = constant.



$$W = \int_{V_1}^{V_2} P.dV.$$

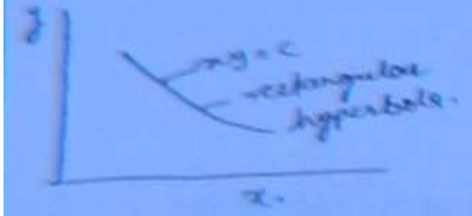
$$W = P\int_{V_1}^{V_2} dV.$$

$$\boxed{W = P(V_2 - V_1)}$$

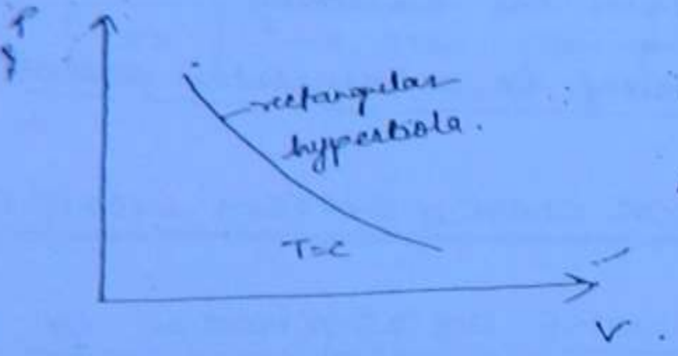
W = Area.

$$\boxed{W = P(V_2 - V_1)} \leftarrow$$

3) CONSTANT TEMPERATURE OR ISOTHERMAL PROCESS :-



$xy = c$



T = constant.

$PV = mRT$

$PV = C.$

$\therefore PV = mRT = C.$

$PV = C \Rightarrow P = C/V.$

$P_1V_1 = P_2V_2 = C.$

Now,

$$W = \int_{V_1}^{V_2} P.dV$$

$$W = \int_{V_1}^{V_2} \frac{C}{V}.dV$$

$$\text{or,}\quad W = C\int_{V_1}^{V_2} \frac{dV}{V}$$

$$W = C\left[\ln V_2 - \ln V_1\right].$$

An adiabatic process can never be an <u>isolated</u> process

$$\text{or,}\quad W = C\ln\left(\frac{V_2}{V_1}\right)$$

$$\therefore\quad \boxed{W = P_1V_1 \ln\left(\frac{V_2}{V_1}\right)}$$

$$\boxed{W = mRT\ln\left(\frac{V_2}{V_1}\right)}$$

((Isothermal curves for an ideal gas are rectangular hyperbola on P-V diagram.))

④ **ADIABATIC PROCESS** :- A process is said to be adiabatic process in which there is no heat transfer from the system or to the system.

for adiabatic process;

$$\boxed{PV^{\gamma} = C}$$

where, γ = adiabatic index and it is greater than 1 ($\gamma > 1$).

$$PV^{\gamma} = C$$

$$P = \frac{C}{V^{\gamma}}$$

$$\boxed{P = C\cdot V^{-\gamma}}$$

$$P_1 = C\cdot V_1^{-\gamma} \quad \text{— ①}$$

$$P_2 = C\cdot V_2^{-\gamma} \quad \text{— ②}.$$

Now

$$W = \int_{V_1}^{V_2} P\,dV$$

$$W = \int_{V_1}^{V_2} C\cdot V^{-\gamma}\,dV$$

$$W = C\int_{V_1}^{V_2} V^{-\gamma}\,dV.$$

$$W = C\left[\frac{V^{-\gamma+1}}{-\gamma+1}\right]_{V_1}^{V_2}$$

$$W = \frac{C}{-\gamma+1}\left[V_2^{-\gamma+1} - V_1^{-\gamma+1}\right]$$ (22)

$$W = \frac{1}{-\gamma+1}\left[\underbrace{C \cdot V_2^{-\gamma}} \cdot V_2 - \underbrace{C \cdot V_1^{-\gamma}} \cdot V_1\right].$$

$$\Rightarrow W = \frac{1}{-\gamma+1}\left[P_2 \cdot V_2 - P_1 \cdot V_1\right]$$

$$\Rightarrow W = \frac{\cancel{-}(P_1V_1 - P_2V_2)}{\cancel{-}\cdot(\gamma-1)}$$

$$\therefore \boxed{W = \frac{(P_1V_1 - P_2V_2)}{(\gamma-1)}}$$

⑤ POLYTROPIC PROCESS :-

A process is said to be a polytropic process, if pressure & volume follows the relation $PV^n = C$, where 'n' is known as polytropic index.

Generally, $\boxed{n > 1 \text{ and } n < \gamma.}$

In polytropic process, there is both heat transfer and work transfer but in adiabatic process, there is only work transfer.

Polytropic Work,

$$\boxed{W = \frac{(P_1V_1 - P_2V_2)}{(n-1)}}$$

$PV^K = C$

Any process can be represented as $PV^K = C$.

for constant Pressure process,

$P = C$.

$PV^K = C$.

㉓

$K = 0$,

$PV^0 = C$

$P \cdot 1 = C$

$\boxed{P = C}$

* for thermal constant process,

$T = C$

$PV = mRT$.

$PV = C$

$PV^1 = C$

$\boxed{K = 1}$

* for constant volume process,

$V = C$

$PV^K = C$.

$(PV^K)^{1/K} = C^{1/K}$

$P^{1/K} \cdot V = C^{1/K}$. (constant new).

when, $K = \infty$

$P^0 \cdot V = C$

$\therefore \boxed{V = C}$

* for adiabatic process,

$PV^\gamma = C$.

$\boxed{K = \infty}$.

* for polytropic process,

$PV^n = C$

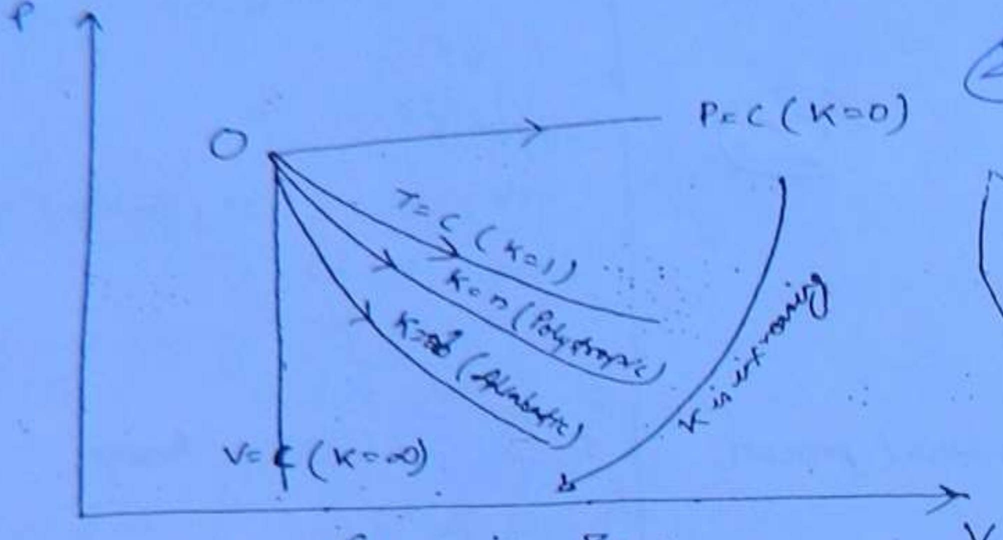
$\boxed{K = n}$

$P = C \rightarrow K = 0$.
$V = C \rightarrow K = \infty$
$T = C \rightarrow K = 1$
Adiabatic $\rightarrow K = \gamma$
Polytropic $\rightarrow K = n$

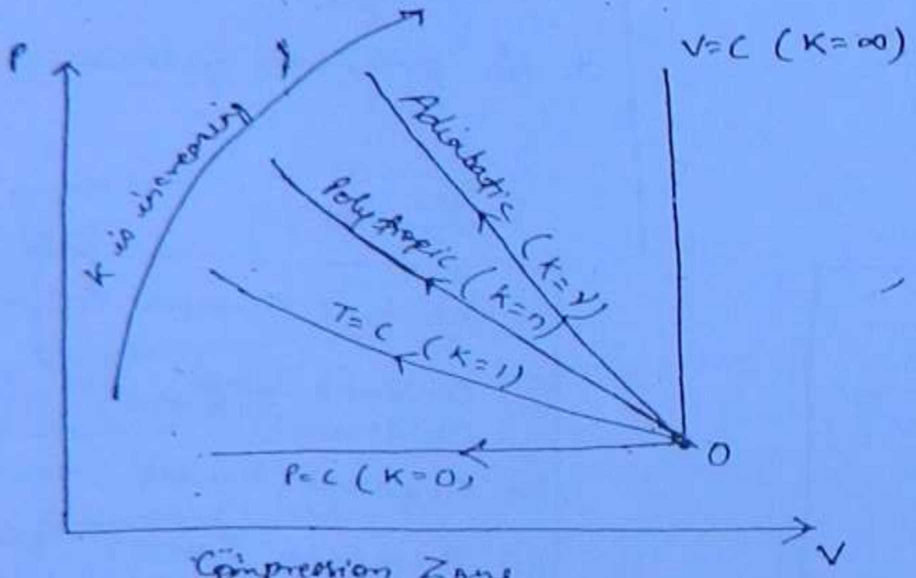
① Constant Pressure, $P = C, \rightarrow K = 0$
② Constant Temp^r, $T = C, \rightarrow K = 1$.
(Isothermal)
③ Polytropic Process, $K = n$.
④ Adiabatic Process $K = \gamma$.
⑤ Constant volume, $V = C \rightarrow K = \infty$.

$PV^\gamma = C$.

⇒ * Representation of Various Process on P-V diagram :-



Expansion Zone



Compression Zone

⇒ * Ideal Gas Equations for Various Processes :-

① Constant Volume Process :-

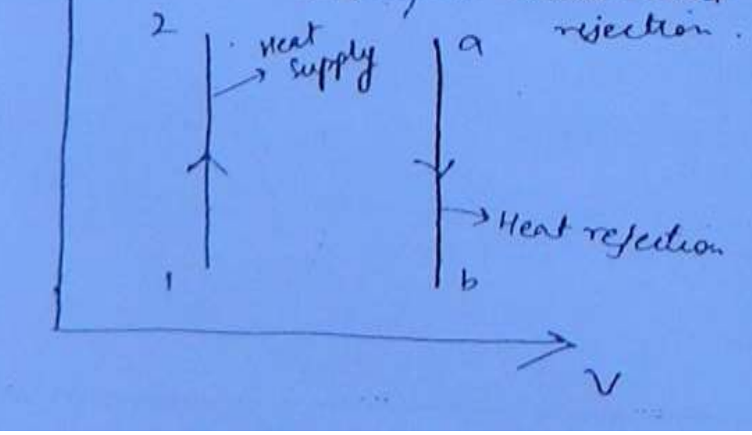
$V = C.$

$P\overset{c}{\cancel{V}} = \overset{c}{\cancel{m}}\overset{c}{\cancel{R}}T$

$P \propto T.$

$$\boxed{\frac{P_2}{P_1} = \frac{T_2}{T_1}}$$

Increase in pressure means heat supply and decrease in pressure means heat rejection.



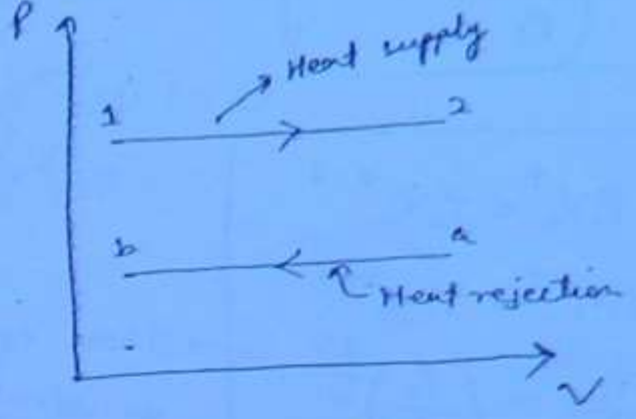
② Constant Pressure Process :-

$P = \text{constant}$

$\overset{c}{\cancel{P}}V = \overset{c}{\cancel{m}}\overset{c}{\cancel{R}}T$

$V \propto T$

$$\boxed{\frac{V_2}{V_1} = \frac{T_2}{T_1}}$$



③ Isothermal Process :-

$T = \text{constant}$

$PV = \overset{c}{\cancel{m}}\overset{c}{\cancel{R}}\overset{c}{\cancel{T}}$

$PV = C$

$$\boxed{P_1V_1 = P_2V_2}$$

④ Adiabatic Process :-

$PV^{\gamma} = c$

$PV = mRT$

$P = \frac{mRT}{V}$

$\frac{mRT}{V} \cdot V^{\gamma} = C$

or, $T \cdot V^{\gamma-1} = \frac{C}{RT} = C$

or, $T \cdot V^{\gamma-1} = \text{const}$

$$\boxed{P_1V_1^{\gamma} = P_2V_2^{\gamma}} \quad -① \quad \Rightarrow \left(\frac{V_1}{V_2}\right) = \left(\frac{P_2}{P_1}\right)^{1/\gamma}$$

$$\boxed{T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}} \quad -② \quad \Rightarrow \left(\frac{V_1}{V_2}\right) = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}}$$

Thus, $\left(\frac{T_2}{T_1}\right)^{1/\gamma-1} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}}$

$$\boxed{\left(\frac{T_2}{T_1}\right) = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}}$$

Thus

$$\boxed{\begin{aligned} P_1V_1^{\gamma} &= P_2V_2^{\gamma} \\ T_1V_1^{\gamma-1} &= T_2V_2^{\gamma-1} \\ \frac{T_2}{T_1} &= \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \end{aligned}}$$

→ These equations are valid for an ideal gas undergoing adiabat process.

5) Polytropic Process :-

$$\boxed{\begin{aligned} P_1V_1^{n} &= P_2V_2^{n} \\ T_1V_1^{\gamma-1} &= T_2V_2^{\gamma-1} \\ \frac{T_2}{T_1} &= \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} \end{aligned}}$$

→ These equations are valid for an ideal gas undergoing ~~adi~~ polytropic process.

* SLOPE OF ISOTHERMAL CURVES ON P-V DIAGRAM :-

$T =$ Constant

$PV = mRT$

$PV = C.$

$PdV + VdP = 0$

$PdV = -VdP$

$\frac{dP}{dV} = -\frac{P}{V}$

Slope of isothermal curves on P-V diagram $= -\frac{P}{V}$

⇒* SLOPE OF ADIABATIC CURVES ON P-V DIAGRAM :-

$$PV^{\gamma} = C$$

$$P[\gamma \cdot V^{\gamma-1} \cdot dV] + V^{\gamma}[dP] = 0 .$$

$$\gamma P \cdot \frac{\cancel{V^{\gamma}}}{V} \cdot dV = -\cancel{V^{\gamma}}\, dP$$

$$\Rightarrow \quad -\frac{\gamma P}{V} = \frac{dP}{dV}$$

$$\text{or} \quad \frac{dP}{dV} = \frac{-\gamma P}{V}$$

$$\therefore \quad \boxed{\frac{dP}{dV} = \gamma\left(-P/V\right)}$$

∴ Slope of adiabatic curves = γ(Slope of Isothermal curves)

$$\frac{\text{Slope of adiabatic}}{\text{Slope of isothermal}} = \gamma$$

$\because \gamma > 1$.

$$\therefore \frac{\text{Slope of adiabatic}}{\text{Slope of isothermal}} > 1.$$

Hence,

$$\boxed{\text{Slope of adiabatic curves} > \text{Slope of isothermal curves}}$$

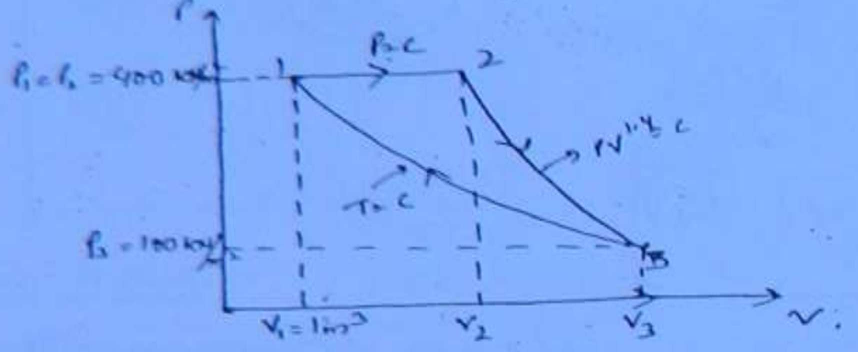
CONVENTIONAL PROBLEM

Q. A system undergoes three processes as shown in figure.

1-2 → Isobaric process
2-3 → Adiabatic process with $\gamma = 1.4$
3-1 → Isothermal process with $PV = C$.

㉘

Find the value of V_2' and also find net work transfer.



Soln.

(Process 3-1)

$T = C$.

Also $PV = C$

$P_3 V_3 = P_1 V_1$

$\Rightarrow 100 \times V_3 = 400 \times 1$

$\therefore V_3 = 4\,m^3$

Again

(Process 2-3)

$PV^{1.4} = C$

$P_2 V_2^{1.4} = P_3 V_3^{1.4}$

$\Rightarrow 400 \times V_2^{1.4} = 100 \times (4)^{1.4}$

$\Rightarrow \boxed{V_2 = 1.486\,m^3}$ (Ans)

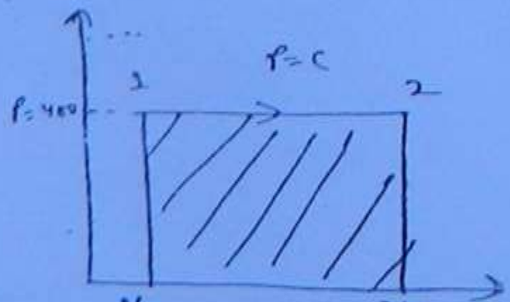
Now.

$W_{net} = W_{12} + W_{23} + W_{31}$

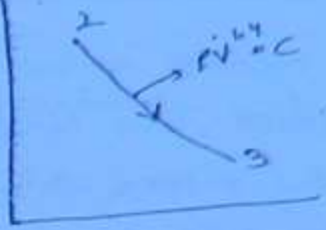
$W_{12} = P(V_2 - V_1)$

$= 400(1.486 - 1)\ \frac{kN}{m^2} \times m^3$

$\therefore W_{12} = 194.4\ kJ$.



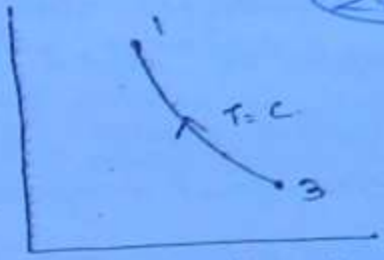
$$W_{23} = \frac{P_2V_2 - P_3V_3}{\gamma - 1}$$



$$\Rightarrow W_{23} = \frac{400 \times 1.486 - 100 \times 4}{1.4 - 1}$$

$$\Rightarrow \boxed{W_{23} = 486\ kJ}$$

28



$$W = P_iV_i \ln \frac{V_f}{V_i}$$

$$W_{31} = P_3V_3 \ln \frac{V_1}{V_3}$$

$$= 100 \times 4 \ln\left(\frac{1}{4}\right)$$

$$\boxed{W_{31} = -554.9\ kJ}$$

$$W_{net} = 194.4 + 486 + (-554.5)$$

$$\therefore \boxed{W_{net} = 125.8\ kJ}$$

<u>Note:-</u> Net work in a cycle is equal to area of the closed region. All clockwise cycles on P-V diagram are work producing or Power Producing ~~for~~ cycles. All anti-clockwise cycles are power absorbing or work absorbing cycles.

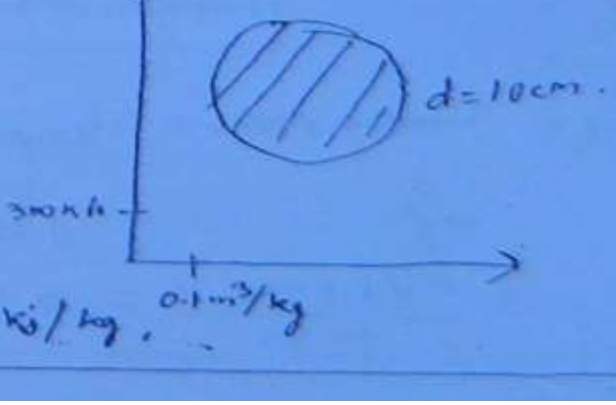
Q:- An imaginary engine recieves heat and performs work on a slowly moving piston at such a rate that the cycle of operation of 1 kg of working fluid can be represented as a circle of 10 cm diameter on P-V diagram, On which 1 cm = 300 kPa and 1 cm = 0.1 m³/kg. Then, find the net work during the cycle.

Soln

1 cm = 300 kPa

1 cm = 0.1 m³/kg

1 cm² = 300×0.1 = 30 kJ/kg

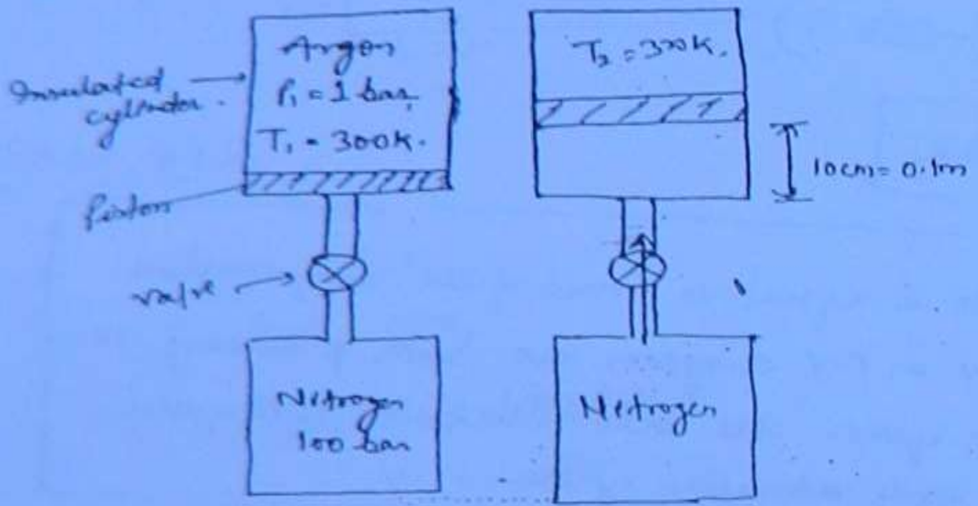


$$W_{net} = Area = \frac{\pi}{4}d^2$$

$$= \frac{\pi}{4}(10)^2 = 78.54\,cm^2$$

$$\therefore W_{net} = (78.54 \times 30) = 2356.2\ kJ/kg.$$

Q. Important An insulated vertical cylinder contain 0.1 kg of Argon gas. With the help of a frictionless non-conducting piston as shown in figure. The mass of the piston is 5 kg and it initially rest on the bottom of the cylinder. The cylinder is connected to nitrogen tank at 100 bar to a pipeline fitted with the valve. The valve is opened and nitrogen slowly enters the cylinder. During this process the piston is lifted to a height of 10 cm by nitrogen gas. The initial pressure and temperature of argon are 300 k and 1 bar. The final temperature of argon gas is 320k. for argon gas $R = 0.208$ kj/kg k and $\gamma = 1.67$.



30

$$W_{argon} = \frac{P_1V_1 - P_2V_2}{\gamma-1} = \frac{m_1R_1T_1 - m_2R_2T_2}{\gamma-1} \quad (\because m \,\&\, R \text{ are constant})$$

$$\therefore W_{argon} = \frac{mR(T_1-T_2)}{\gamma-1}$$

$$= \frac{0.1 \times 0.208\,(300-320)}{1.67-1}$$

$$\therefore \boxed{W_{argon} = -0.621 \text{ kj}}$$

$$W_{N_2} = \underbrace{W_{raising\ piston}}_{mgh} + \underbrace{W_{compressing\ Argon}}_{0.621\ kj}$$

$$= \frac{5 \times 9.81 \times 0.1}{1000} = 4.905 \times 10^{3} \text{ KJ}.$$

J.

∴ Net work done by Nitrogen.

$W_{N_2} = 4.905 \times 10^{-3}\ \text{kJ} + 0.621\ \text{kJ}$

$\boxed{W_{N_2} = 0.626\ \text{kJ}}$ (Ans)

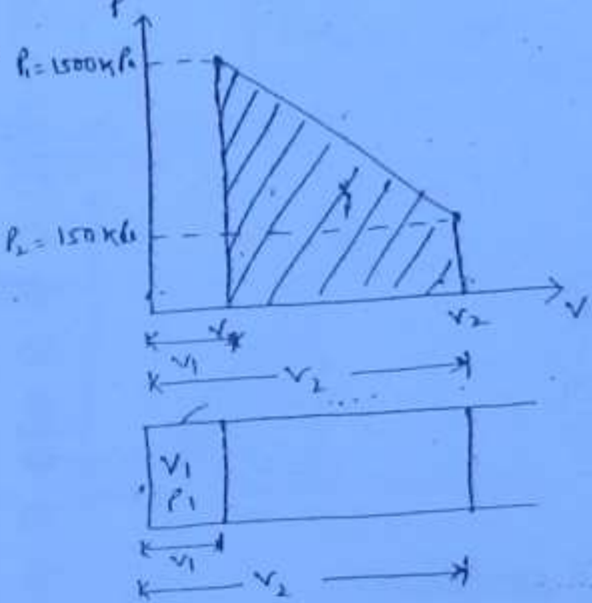
$\boxed{V_s = A \times L}$

V_s = swept vol^m.
L = stroke length
A = Area of Piston.

Q!. Important

An engine cylinder has a Piston area of $0.12\ m^2$ and contains a gas at a pressure of 1.5 MPa. The gas expands according to a process which is represented by a straight line on P-V diagram. The final pressure is 0.15 MPa. Calculat the work done by the gas if the stroke length is 0.3 m.

Sol^n.



Work = Area under curve

$= \frac{1}{2}(1500 + 150) \times (V_2 - V_1)$

But, $V_2 - V_1 = V_s = AL$.

$= \frac{1}{2} \times 1650 \times (0.12 \times 0.3)$

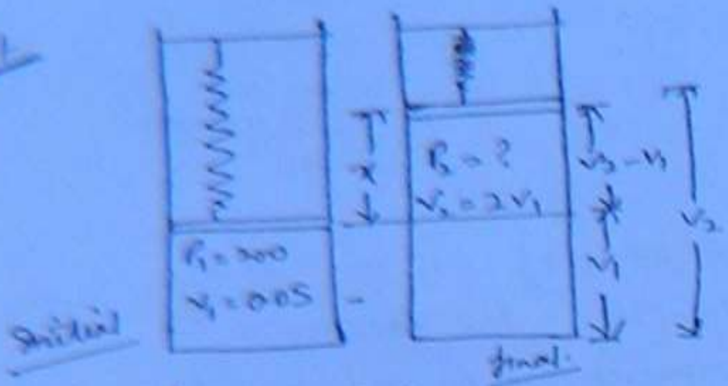
$\therefore \boxed{W = 29.7\ \text{kJ}}$ (Ans)

***
Q!. V.V.Imp

A piston cylinder device contains $0.05\ m^3$ of a gas initially at 200 kPa. At this state a linear spring which has spring constant of 150 kN/m is just touching the piston but exerting no force on it. Now Heat is transferred to the gas causing the piston to rise and to compress the spring until the volume inside the cylinder doubles. If the cross-sectional area of the piston is $0.25\ m^2$, find :-

① final pressure inside the cylinder
② Work done by the gas

Solⁿ:



$V_2 - V_1 = A \times x$

$\Rightarrow 2V_1 - V_1 = 0.25 \times x$

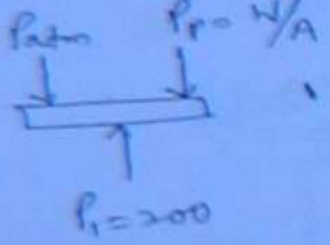
$\Rightarrow 0.05 = 0.25x$

$\therefore \boxed{x = 0.2m}$

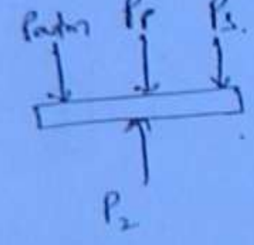
B1

Now free-body diagrams.

Initial



final.



At equilibrium.

$P_{atm} + W/A = ~~P_1~~ 200$ — ①

At equilibrium

$P_{atm} + P_P + P_s = P_2$ — ②

But from eqⁿ ①, $P_{atm} + P_P = 200$

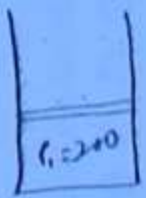
$\therefore 200 + P_s = P_2$ — ③.

Now $P_s = \dfrac{F_s}{A} = \dfrac{K \cdot x}{A}$.

$\therefore P_s = \dfrac{150 \times 0.2}{0.25} = 120$ kN/m² or kPa.

$\therefore \boxed{P_2 = 200 + 120 = 320 \, kPa}$ ... (Ans)

Work done by gas = work done in raising piston + work done in compressing spring.



㉝

Spring Work

$= \frac{1}{2} K x^2$

$= \frac{1}{2} \times 150 \times 0.2^2$

$= 3\ kJ$

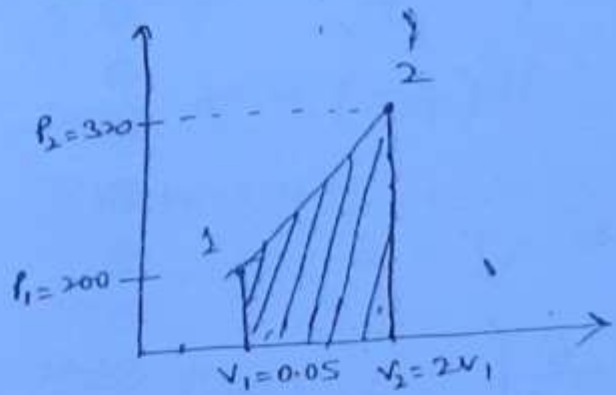
$W_{piston} = P(V_2 - V_1)$

$200(2V_1 - V_1)$

$= 200 \times (0.05) \Rightarrow W_{piston} = 10\ kJ$

$\boxed{W_{Total} = 10+3 = 13\ kJ}$ (Ans)

Alternate Method :



$P_2 = 200 + P_s$.

$P_s = \frac{F_s}{A} = \frac{Kx}{A}$ —①

Also, $V_2 - V_1 = Ax$

$\Rightarrow x = \frac{V_2 - V_1}{A} = \frac{\Delta V}{A}$.

$\therefore P_s = \frac{K}{A} \cdot \frac{\Delta V}{A}$

or $P_s = \frac{K}{A^2} \cdot \Delta V$.

Thus, $P_2 = 200 + \frac{K}{A^2} \cdot \Delta V$.

$W = \frac{1}{2}(200 + 320)(2V_1 - V_1)$

$= \frac{1}{2} \times 520 \times 0.05$

$\therefore \boxed{W = 13\ KJ}$ (Ans)

WORK BOOK

P+F – CFL
2+ ② 27. 30

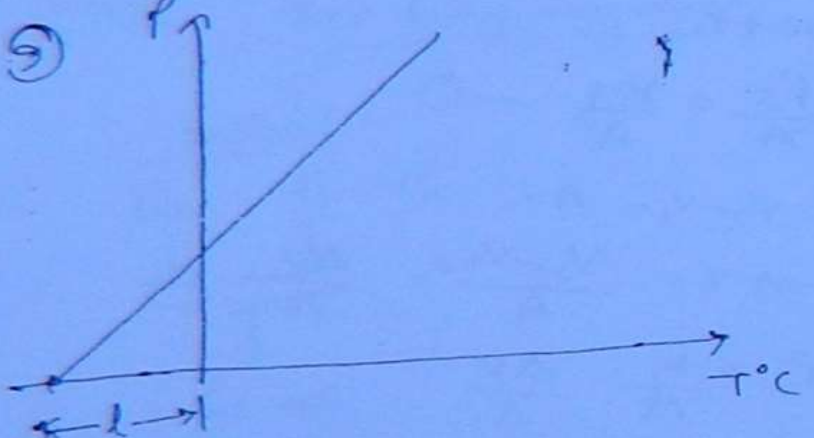
Chapter – ①

① — ⓐ
② — ⓒ
③ — ⓑ
④ — ⓓ
⑤ — ⓐ
⑥ — ⓑ
⑦ — ⓒ
⑧ — ⓒ
⑨ — ⓓ
⑩ — ⓒ
⑪ — ⓑ
⑫ — ⓒ
⑬ — ⓓ
⑭ — ⓓ
⑮ — ⓓ
⑯ — ⓒ
⑰ — ⓐ
⑱ — ⓒ
⑲ — ⓓ
⑳ — ⓓ

38

1-2-7
3-8

⑨



$PV = mRT$.

$P = \frac{mR}{V} \cdot T$, Here, m, R & V are constants.

$\therefore P = CT$.

$\because P = 0$. $\therefore \underline{T = 0K}$.

converting in °C,

$K = °C + 273.15$

$\therefore 0 = °C + 273.15$

$\Rightarrow °C = -273.15$, Hence (d) is ans.

Ans (13) (a) $\int P dv$ & (b) $\int v dP$.

Both are works and work ~~could be~~ is not a property.

$$\boxed{\begin{aligned} dz &= Mdx + Ndy. \\ \left(\frac{\partial M}{\partial y}\right)_x &= \left(\frac{\partial N}{\partial x}\right)_y. \end{aligned}}$$

3a

The equation $dz = Mdx + Ndy$ is exact, when

$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y.$$

Solving option (d) :-

$$\frac{dT}{T} - \frac{vdP}{T}$$

$$\frac{1}{T} \cdot dT + \left(-\frac{v}{T}\right) dP.$$

$$Mdx + Ndy.$$

$$M = \frac{1}{T}; \quad x = T; \quad N = -\frac{v}{T}; \quad y = P.$$

Now, $$\left(\frac{\partial M}{\partial y}\right)_x = \left[\frac{\partial}{\partial P}\left(\frac{1}{T}\right)\right]_T \qquad \therefore \left(\frac{\partial M}{\partial y}\right)_x = 0.$$

$\because$ T is constant

Again, $$\left(\frac{\partial N}{\partial x}\right)_y = \left[\frac{\partial}{\partial T}\left(-\frac{v}{T}\right)\right]_P.$$

from ideal gas equation

$$PV = mRT \quad \therefore \frac{V}{T} = \frac{mR}{P}.$$

$$\therefore \left(\frac{\partial N}{\partial x}\right)_y = \left[\frac{\partial}{\partial T}\left(-\frac{mR}{P}\right)\right]_P.$$

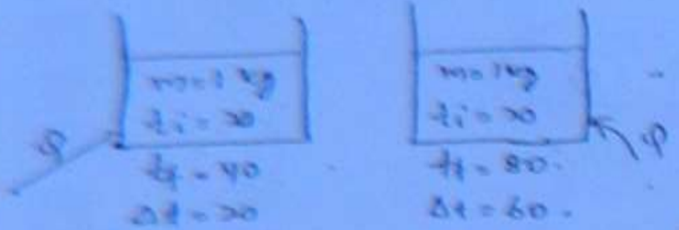
$\because$ P is constant $\therefore \left(\frac{\partial N}{\partial x}\right)_y = 0$

$$\therefore \left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y = 0$$

Hence, these are exact differentials. So, it is a property.

← : HEAT : →

The energy transfer due to temperature difference is known as heat transfer.



$Q \propto \Delta t$

$Q \propto m$

$Q \propto m(\Delta t)$

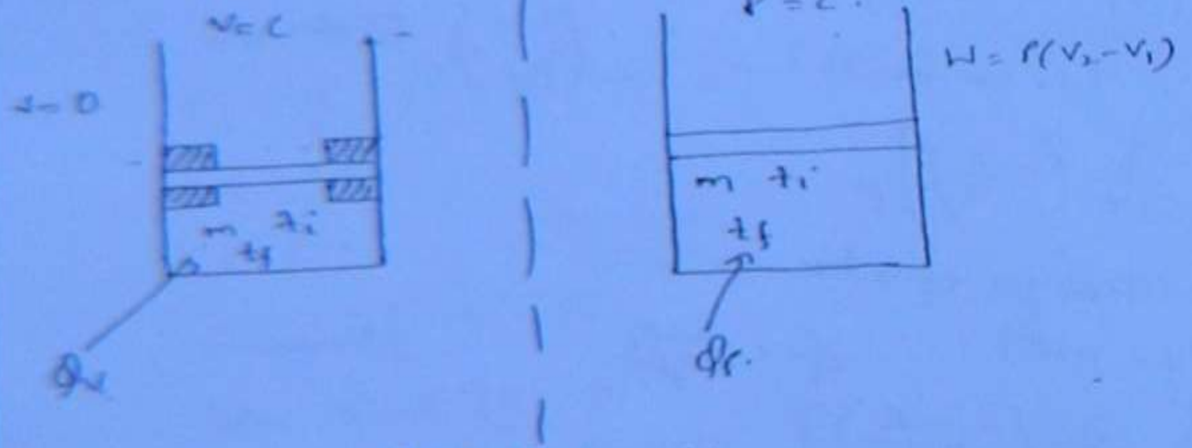
or, $Q = m c (\Delta t)$ → (specific heat)

when $m = 1$; $\Delta t = 1$

$\boxed{Q = c}$

$$\boxed{c = \frac{Q}{m \Delta t} \quad \text{Units} \left(\frac{kJ}{kg\,K}\right)}$$

Specific heat (c) :→ It is the amount of heat required to raise the temperature of unit mass of substance through unit degree temperature difference.



$Q_P > Q_V$

$\cancel{m} C_P \cancel{\Delta t} > \cancel{m} C_V \cancel{\Delta t}$

$\therefore \boxed{C_P > C_V}$

$\because \gamma = \frac{C_P}{C_V}$ ~~as~~. & $\frac{C_P}{C_V} > 1$

$\therefore \boxed{\gamma > 1}$

Specific heat at constant pressure (C_p) is greater than specific heat at constant volume (C_v) because 'C_p' includes internal energy and external work whereas 'C_v' includes internal energy only.

FIRST LAW OF THERMODYNAMICS :→ (59)
(Law of Conservation of Energy) :-

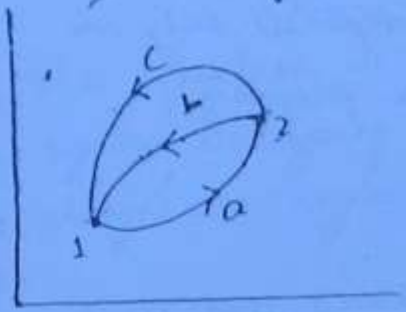
Statement :- for a closed system undergoing a cycle net heat transfer (ΣQ) is equal to net work transfer (ΣW)

$$\boxed{\Sigma Q = \Sigma W}$$

↳ valid for a cycle.

(Results)
Consequences of first law of Thermodynamics :-

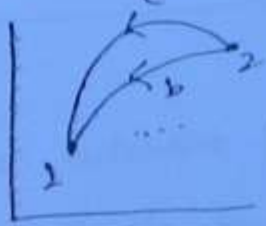
① Heat transfer is a path function.



$$\cancel{(\text{đ}Q)_{1a2}} + (\text{đ}Q)_{2b1} = \cancel{(\text{đ}W)_{1a2}} + (\text{đ}W)_{2b1}$$

$$\underline{\cancel{(\text{đ}Q)_{1a2}} + (\text{đ}Q)_{2c1} = \cancel{(\text{đ}W)_{1a2}} + (\text{đ}W)_{2c}}$$

$$(\text{đ}Q)_{2b1} - (\text{đ}Q)_{2c1} = (\text{đ}W)_{2b1} - (\text{đ}W)_{2c}$$



$$(\text{đ}W)_{2b1} \neq (\text{đ}W)_{2c1}$$

$$\therefore (\text{đ}W)_{2b1} - (\text{đ}W)_{2c1} \neq 0.$$

$$\therefore (\text{đ}Q)_{2b1} - (\text{đ}Q)_{2c1} \neq 0.$$

Hence, $(\text{đ}Q)_{2b1} \neq (\text{đ}Q)_{2c1}$

Though for paths 'b' and 'c', end points are same but heat transfer is not same and hence heat transfer is a path function.

* Important points with respect to heat transfer and work transfer :-

1) Both are path functions.

2) Both are not properties.

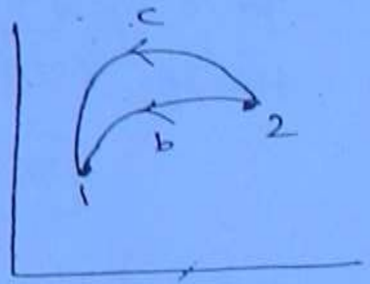
) Both are inexact differentials.

) Both are boundary phenomenon. ㊳

$(\text{đ} = \delta)$

nsequences continued....

- Energy is a property.



$$\text{đ}Q_{2b1} - \text{đ}Q_{2c1} = (\text{đ}W)_{2b1} - (\text{đ}W)_{2c1}$$

$$\text{đ}Q_{2b1} - \text{đ}W_{2b1} = \text{đ}Q_{2c1} - \text{đ}W_{2c1}$$

$$\boxed{(\text{đ}Q - \text{đ}W)_{2b1} = (\text{đ}Q - \text{đ}W)_{2c1}}$$

~~e quantity~~ The quantity $(\text{đ}Q - \text{đ}W)$ is same for paths 'b' and 'c' and hence it does not depend on path and it depends only on end points. Therefore, $(\delta Q - \delta W)$ must be a property. and this property is known as energy.

$$(\text{đ}Q - \text{đ}W)_{2b1} = (\text{đ}Q - \text{đ}W)_{2c1} = dE$$

$$(\delta Q - \delta W)_{2b1} = dE$$

$$\boxed{\delta Q = dE + \delta W}$$

his is the first law of thermodynamics for a closed stem undergoing a process (reversible or irreversible).

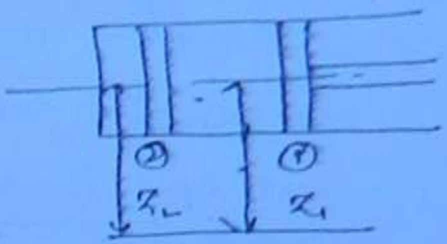
$\boxed{\text{đ}Q = dE + PdV}$ It is the first law of hermodynamics for a closed system undergoing reversible rocess because ~~PdV~~ '$W = PdV$' is valid for a reversible process.

[redacted]

Now,

$$E = \underbrace{K.E + P.E}_{\text{Macrosciopic}} + U \leftarrow \text{Microscopic}$$

$$dE = d(KE) + d(PE) + dU.$$



Let The system is initially ~~and~~ final at rest, so K.E = 0 and again finally it is also at rest, ∴ KE = 0 again.

The height is also same ∴ change in Potential Energy. $d(PE) = 0$.

$$\therefore dE = dU.$$

Now, $dQ = dE + dW$

$$\therefore \boxed{dQ = dU + dW.}$$

This equation is first law of thermodynamics equation for a system undergoing a process when kinetic and potential energy changes are neglected.

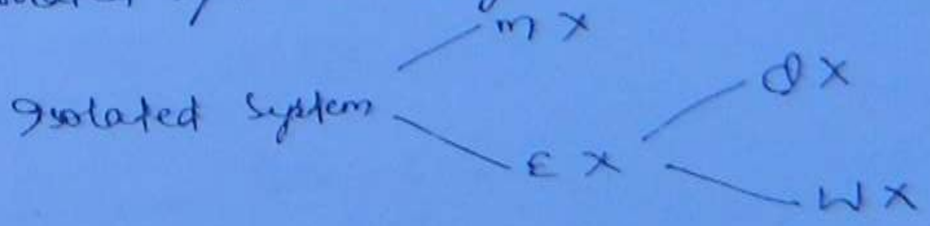
INTERNAL ENERGY (U) :-

It is the energy associated with molecules. is an extensive property. and it is generally expressed in kJ and it is an intensive property.

Specific Internal Energy $= U/m$

It is generally expressed in kJ/kg.

③ Energy of an isolated system is always constant.



* To show that "$(C_p - C_v) = R$" for an ideal gas :-

(Meyer's Eqⁿ):

$$H = U + PV$$ ①

$$dH = dU + d(PV)$$

$$\cancel{m}c_p \cancel{dT} = \cancel{m}c_v \cancel{dT} + d(mRT)$$ (for an ideal gas)

$$\Rightarrow \cancel{m}c_p \cancel{dT} = \cancel{m}c_v \cancel{dT} + \cancel{m}R \cancel{dT}$$

$$\Rightarrow c_p = c_v + R.$$

or $\boxed{C_p - C_v = R}$ → Meyer's Equation

Now

$$\frac{C_p}{C_v} = \gamma \Rightarrow C_p = \gamma . C_v.$$

$$\therefore \gamma C_v - C_v = R$$

$$\Rightarrow \boxed{C_v = \frac{R}{\gamma - 1}} \text{ —— ②}$$

$$\therefore C_p = \gamma . C_v$$

$$\therefore \boxed{C_p = \frac{\gamma . R}{\gamma - 1}} \text{ —— ③}$$

for Air

$C_p = 1.005$ kj/kg k

$C_v = 0.718$ kj/kg k

$R = 0.287$ kj/kg k

$\gamma = 1.4.$

③ Isothermal Process :-

$$dQ = dU + dW$$

for an ideal gas , $U = f(T)$.

Here, $T =$ constant

$\therefore U =$ constant $\Rightarrow dU = 0$ ㊵

$$\therefore \quad dQ_T = 0 + dW$$

$$\Rightarrow \boxed{dQ_T = dW}$$

When an ideal gas undergoes isothermal process, heat transfer is equal to work transfer.

④ Adiabatic Process :-

~~$dQ = dU + dW$~~ $Q = 0$.

There is no heat transfer in adiabatic process.

⑤ Polytropic Process :-

$$dQ = dU + dW$$

for an ideal gas, $dU = mc_v dT$.

& Polytropic work $= dW = \left(\frac{P_1V_1 - P_2V_2}{n-1}\right)$

$$\therefore \quad dQ = mc_v dT + \frac{P_1V_1 - P_2V_2}{(n-1)}$$

$$= m.\frac{R}{\gamma-1} dT + \frac{P_1V_1 - P_2V_2}{n-1}$$

$$= \frac{mR}{\gamma-1}(T_2 - T_1) + \frac{P_1V_1 - P_2V_2}{n-1}$$

$\because mRT = PV. \quad \therefore mRT_2 = P_2V_2$ & $mRT_1 = P_1V_1$

$$\therefore \quad dQ = \frac{P_2V_2 - P_1V_1}{\gamma-1} + \frac{P_1V_1 - P_2V_2}{n-1}$$

$$= \frac{P_1V_1 - P_2V_2}{n-1} + \frac{P_1V_1 - P_2V_2}{n-1}$$

$$\Rightarrow \quad dQ = P_1V_1 - P_2V_2 \left[\frac{1}{n-1} - \frac{1}{\gamma-1} \right].$$

$$= P_1V_1 - P_2V_2 \left[\frac{\gamma-1-(n-1)}{(n-1)(\gamma-1)} \right].$$

$$= \frac{P_1V_1 - P_2V_2}{n-1} \left[\frac{\gamma-n}{\gamma-1} \right].$$ ㊷

$$\therefore \boxed{dQ = \left(\frac{\gamma-n}{\gamma-1}\right) W_{polytropic}}$$ VVI

Polytropic Specific heat (C_{poly}) :-

$$dQ_v = dU = mC_v dT \quad \text{(for ideal gas)}$$

$$dQ_v = mC_v dT \quad \text{— ①}$$

$$dQ_p = dH = mC_p.dT \quad \text{(for ideal gas).}$$

$$dQ_p = mC_p dT \quad \text{— ②}$$

Now for polytropic process!

$$dQ_{poly} = \left(\frac{\gamma-n}{\gamma-1}\right) \times \left(\frac{P_1V_1 - P_2V_2}{n-1}\right).$$

$$\because PV = mRT.$$

$$\therefore \quad dQ_{poly} = \frac{(\gamma-n)}{(\gamma-1)} \left[\frac{mRT_1 - mRT_2}{n-1} \right].$$

$$= \frac{(\gamma-n)}{(\gamma-1)} \frac{mR}{n-1} (T_1 - T_2)$$

$$= \frac{(\gamma-n)}{(\gamma-1)} \cdot \frac{mR}{(n-1)} (-dT).$$

$$\because (dT = T_2 - T_1) \quad \text{Also, } \frac{R}{\gamma-1} = C_v.$$

$$\therefore \quad dQ_{poly} = \frac{(\gamma-n)}{(n-1)} mC_v(-dT).$$

or $$dQ_{poly} = \frac{(n-\gamma)}{(n-1)} \cdot mc_v(dT).$$

Rearranging above equation.

$$\boxed{dQ_{poly} = m\left[\frac{n-\gamma}{n-1} C_v\right] dT.}$$

$$\therefore \boxed{C_{poly} = \frac{n-\gamma}{n-1} \cdot C_v.}$$

for '$n>1$' and '$n<\gamma$'; 'C_{poly}' is negative i.e. though heat is supplied, there is a decrease in temperature. This is because in such a polytropic process, work transfer is more than heat transfer and this excess work transfer comes from internal energy of the system and as there is a decrease in internal energy & the temperature also decreases.

⇒* To show that PV^γ = constant for adiabatic Process :-

$$dQ = dU + dW$$

for reversible process.

$$dQ = dU + PdV.$$

Ideal gas undergoing adiabatic process.

$$dU = mc_v dT, \quad \because dQ = 0$$

$$\therefore 0 = mc_v dT + PdV$$

$$\text{or, } PdV = -mc_v dT \quad \text{——} ①$$

Now $$H = U + PV$$

$$\rightarrow dH = \underbrace{dU + PdV}_{dQ} + VdP$$

$$\therefore dH = dQ + VdP$$

for ideal gas :-

$$dH = mc_p dT$$

$$\text{adiabatic} \Rightarrow dQ = 0.$$

$$\therefore mc_p dT = 0 + VdP.$$

$$\Rightarrow mc_p dT = VdP \quad \text{—} \; ②.$$

(4B)

Dividing eqⁿ ① by eqⁿ ② :-

$$\frac{\cancel{m}c_p \cancel{dT}}{-\cancel{m}c_v \cancel{dT}} = \frac{VdP}{PdV}$$

$$\Rightarrow -\gamma = \frac{V}{dV} \cdot \frac{dP}{P}.$$

$$\Rightarrow -\gamma . \frac{dV}{V} = \frac{dP}{P}.$$

$$\Rightarrow \frac{dP}{P} + \frac{\gamma dV}{V} = 0.$$

$$\Rightarrow \ln P + \gamma \ln V = \ln C$$

$$\Rightarrow \ln P + \ln V^\gamma = \ln C$$

$$\Rightarrow \ln PV^\gamma = \ln C.$$

$$\therefore \boxed{PV^\gamma = C}$$

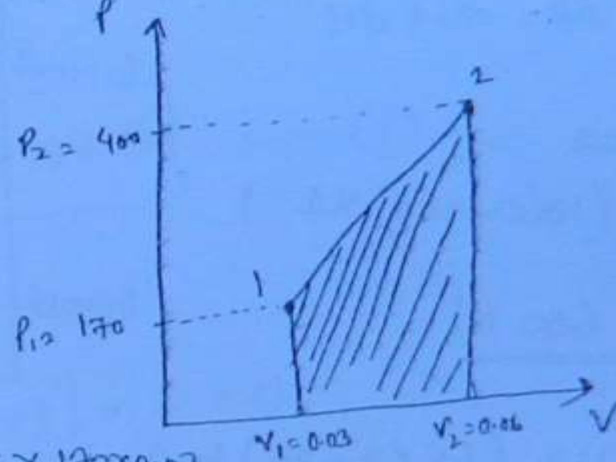
The equation "$PV^\gamma = C$" is valid for an ideal gas undergoing <u>reversible</u> adiabatic process.

-: CONVENTIONAL QUESTIONS :-

IAS
40 marks

Q:- A fluid is contained in a cylinder by spring loaded friction piston so that pressure in the fluid is a linear function of volume i.e. $P = a + bV$, where 'a' and 'b' are constants. Internal energy of the fluid is given by $U = 34 + 3.15\,PV$, where 'U' is in ~~kilo~~ kJ, 'P' in kPa and 'V' in m^3. If the fluid changes from initial state of $P_1 = 170$ kPa, $V_1 = 0.03\ m^3$ to a final state of $P_2 = 400$ kPa, $V_2 = 0.06\ m^3$, find the magnitude and direction of heat transfer and work transfer. [illegible]

Sol^n :-



$W =$ Area under curve

$= \frac{1}{2} \times (170 + 400) \times 0.03$

$=$

$dU = U_2 - U_1 = 3.15\,P_2V_2 - 3.15\,P_1V_1$

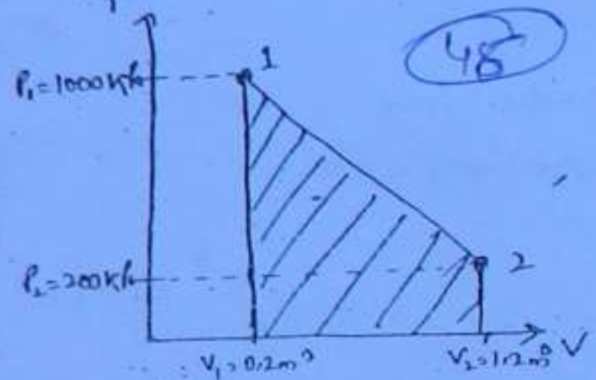
$= 3.15 \times 400 \times 0.06 - 3.15 \times 170 \times 0.03$

$\therefore dU =$

$dQ = dU + dW$

Q. A gas of mass 1.5 kg undergoes a reversible expansion process which follows the relationship $P = a + bV$, where 'a' and 'b' are constants. The initial and final pressures are 1000 kPa and 200 kPa respectively and the corresponding volumes are ~~B~~ $V_1 = \overset{0.2}{\cancel{0.02}}\ m^3$ and $V_2 = 1.2\ m^3$. The specific internal energy, 'u' $= (1.5\,Pv - 85)$ kJ/kg where 'P' is in kPa and small 'v' is m^3/kg. Calculate net heat transfer and maximum internal energy of the gas.

P.1 $dQ = dU + dW$



48

$W = $ Area
$= \frac{1}{2}(1000 + 200) \times 1$

$W = 600$ kJ.

Now, $u = (1.5\,Pv - 85)$ kJ/kg

$u = U/m \Rightarrow U = u \times m.$

$\therefore U = m[1.5\,Pv - 85]$

$= [1.5\,Pvm - m.85]$

$v = \frac{V}{m} \Rightarrow V = v.m.$

$\therefore U = (1.5\,PV - 1.5 \times 85)$ $\quad \because (m = 1.5\ kg)$

$$\therefore \boxed{U = (1.5\,PV - 127.5)}$$

$dU = U_2 - U_1$

Now $U_2 = 1.5\,P_2V_2 - 127.5$

$U_1 = \underline{1.5\,P_1V_1 \mp 127.5}$

$dU = U_2 - U_1 = 1.5(P_2V_2 - P_1V_1)$

$\therefore dU = 1.5[200\times 1.2 - 1000\times 0.2]$

$= 60\ \text{kJ}.$

Now, $dQ = dU + dW$

$= (60+600)\ \text{kJ}$

$\therefore \boxed{dQ = 660\ \text{kJ}}$ (Ans)

Now, 2nd Part :-

$U = 1.5PV - 127.5.$ (47)

$\because P = a + bV$

$\therefore U = 1.5(a+bV)V - 127.5$

$\Rightarrow \boxed{U = 1.5[aV + bV^2] - 127.5}$ —— ①

for max^m U, $\frac{dU}{dV} = 0.$

$\frac{dU}{dV} = 1.5[a(1) + b(2V)] = 0 = 0$

$\Rightarrow 1.5[a + 2bV] = 0.$

$\therefore (a + 2bV) = 0 \Rightarrow \boxed{V = \frac{-a}{2b}.}$

or, $a = -2bV$

$P_1 = a + bV_1 \Rightarrow 1000 = \cancel{a} + b\times 0.2$

$P_2 = a + bV_2 \Rightarrow \underline{-200 = \cancel{a} + b\times 1.2}$

$800 = -b$

or $\boxed{b = -800}$

Again

$200 = a + 1.2b \Rightarrow$ ~~$a = 200 - 1.2\times(-800)$~~

$\Rightarrow a = 200 - 1.2(-800)$

$\therefore \boxed{a = 1160}$

Now $V = \frac{-1160}{2\times(-800)} \Rightarrow V = 0.725\,m^3$.

$$U = 1.5\left[1160 \times 0.725 + (-800)\,0.725^2\right] - 127.5$$

$$\therefore \boxed{U_{max} = 503.25\ kJ.}$$ (Ans) (48)

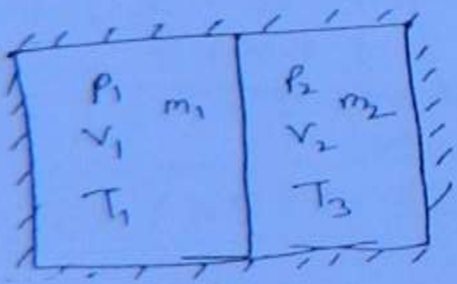
***

Q:- 11 An insulated rigid pressure vessel is divided into two portions by a thin partition. First part of vessel is occupied by an ideal gas at a pressure 'P_1', volume 'V_1' and temperature 'T_1'. The other part is occupied by the same ideal gas but at a pressure 'P_2', volume 'V_2' and temperature 'T_2'. Suddenly the partition is removed and two portions mix with each other. Show that the final pressure P_3 and final temp^n 'T_3' are given by

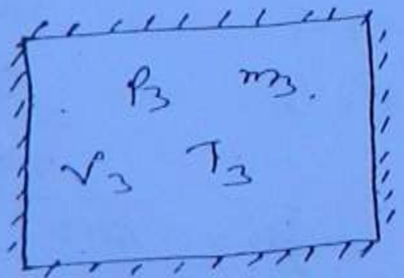
$$P_3 = \frac{P_1V_1 + P_2V_2}{V_1 + V_2}$$

$$T_3 = \frac{P_1V_1 + P_2V_2}{\frac{P_1V_1}{T_1} + \frac{P_2V_2}{T_2}}$$

Ans:- $PV = mRT \Rightarrow m = \frac{PV}{RT}$



Initial



final.

$$m_3 = m_1 + m_2$$

$$\frac{P_3V_3}{RT_3} = \frac{P_1V_1}{RT_1} + \frac{P_2V_2}{RT_2}$$

$$\frac{P_3V_3}{T_3} = \left(\frac{P_1V_1}{T_1} + \frac{P_2V_2}{T_2}\right)$$

$$\Rightarrow T_3 = \frac{P_3V_3}{\left(\frac{P_1V_1}{T_1} + \frac{P_2V_2}{T_2}\right)} \quad \text{——} ①$$

(48)

Now $dQ = dU + dW$.

for constant vol^m, closed system, $dW = 0$.

Also, ∵ the system is insulated

$\therefore dQ = 0$.

$$\therefore 0 = dU + 0$$

$$\Rightarrow dU = 0$$

$$\Rightarrow U_f - U_i = 0 \quad \Rightarrow U_f = U_i$$

Now $U_3 = U_1 + U_2$

$$m_3 C_v T_3 = m_1 C_v T_1 + m_2 C_v T_2$$

$$\Rightarrow \frac{m_3 R T_3}{\cancel{\gamma-1}} = \frac{m_1 R T_1}{\cancel{\gamma-1}} + \frac{m_2 R T_2}{\cancel{\gamma-1}}$$

$$\Rightarrow P_3V_3 = P_1V_1 + P_2V_2. \quad \text{——} ②$$

Also, $V_3 = V_1 + V_2$

$$\therefore P_3(V_1+V_2) = P_1V_1 + P_2V_2$$

$$\Rightarrow \boxed{P_3 = \frac{P_1V_1 + P_2V_2}{V_1+V_2}}$$ Proved

& $$\boxed{T_3 = \frac{P_1V_1 + P_2V_2}{\left(\frac{P_1V_1}{T_1} + \frac{P_2V_2}{T_2}\right)}}$$ Proved

from eq^n (2)

Q: A fluid system contained in a piston-cylinder machine passes through a complete cycle of four processes. The summation of heat transfer is -340 kJ/cycle. The system completes 200 cycles per minute. Complete the following table and also find the net work transfer in Kwatt.

Process	Q (kj/min)	W (kj/min)	dU (kJ/min)
1-2	0	4340	-4340
2-3	42000	0	42000
3-4	-4200	69000	-73200
4-1	-105800	-141340	35540.

Soln: For (Process - 1-2) :-

$dQ = dU + dW$

$0 = dU_{12} + 4340$

$\Rightarrow dU_{12} = -4340.$

For 2-3 (Process):-

$dQ = dU + dW$

$42000 = dU + 0 \Rightarrow dU_{23} = 42000$

For 3-4 (Process):-

$dQ = dU + dW$

$\Rightarrow -4200 = dW_{34} \pm 73200$

$\Rightarrow dW_{34} = 69000.$

Now,

$\Sigma Q = -340$ kj/cycle.

& There are 200 cycles/minute.

$\therefore \Sigma Q = -340 \times 200 = -68000$ kj/min.

Also, $\Sigma Q = Q_{12} + Q_{23} + Q_{34} + Q_{41}$

$\Rightarrow -68000 = 0 + 42000 + (-4200) + Q_{41}$

$\Rightarrow \underline{Q_{41} = -105800.}$

Again, $\Sigma Q = \Sigma W = -68000.$

$\Sigma W = W_{12} + W_{23} + W_{34} + W_{41}$

$\Rightarrow W_{41} = -68000 - 4340 - 0 - 69000$

$\therefore \underline{W_{41} = -141340}$

Now, for Process $\underline{(4\text{-}1)}$

$dQ = dU + dW$

$\Rightarrow dU = dQ - dW$

$= -105800 + (-141340)$

$\Rightarrow dU_{41} = \underline{\underline{35540}}.$

(56).

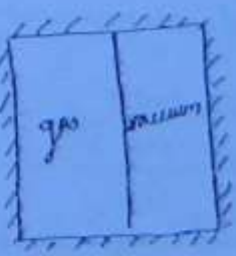
Now, $\dot{W}_{net} = \Sigma W = -68000$ kj/min.

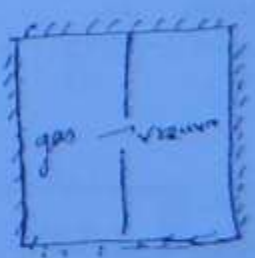
$\dot{W}_{net} = -\frac{68000}{60}$ kj/sec = kW.

$$\boxed{\dot{W}_{net} = -1133.3 \text{ KW}}$$ (Ans)

THEORY

⇒* **FREE EXPANSION** :→ The expansion of a gas against vacuum is known as free expansion.





free expansion or Restrained expansion

Free expansion work is equal to zero because as the gas is expanding against vacuum, there is ~~not~~ no resistance offered from surrounding and hence work is equal to zero.

(52)

$dQ = dU + dW$

$0 = dU + 0$

$\Rightarrow dU = 0$

$U_f - U_i = 0 \Rightarrow U_f = U_i$

If an ideal gas undergoes free expansion

$\downarrow$

$U = f(T)$

$\Rightarrow \boxed{U_i = U_f \Rightarrow T_i = T_f}$

Also,

$\therefore H = \cancel{P}U + PV$

$H = f(T) + \overset{c}{m}\overset{c}{R}T$

$\therefore H = \phi(T)$.

As, $\boxed{T_i = T_f \Rightarrow H_i = H_f}$.

Note:- When an ideal gas undergoes free expansion :-

(i) $U_i = U_f$

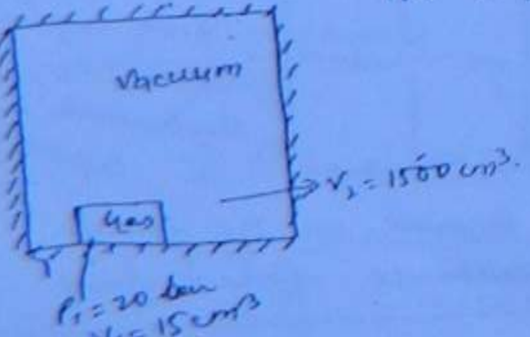
(ii) $T_i = T_f$

(iii) $H_i = H_f$.

PROBLEM

Q:- An ideal gas at 20 bar and 40°C is contained in a small cylinder having a volume of 15 cm³. This cylinder is placed inside a large container having a volume of 1500 cm³. The large container is perfectly insulated and evacuated. By an appropriate means, the gas is allowed to discharge and fill the large container. Find the final pressure after the entire assembly reaches the equilibrium.

Sol



$T_1 = T_2$ (for free expansion).

$PV = mRT$.

$m = \frac{PV}{RT}$

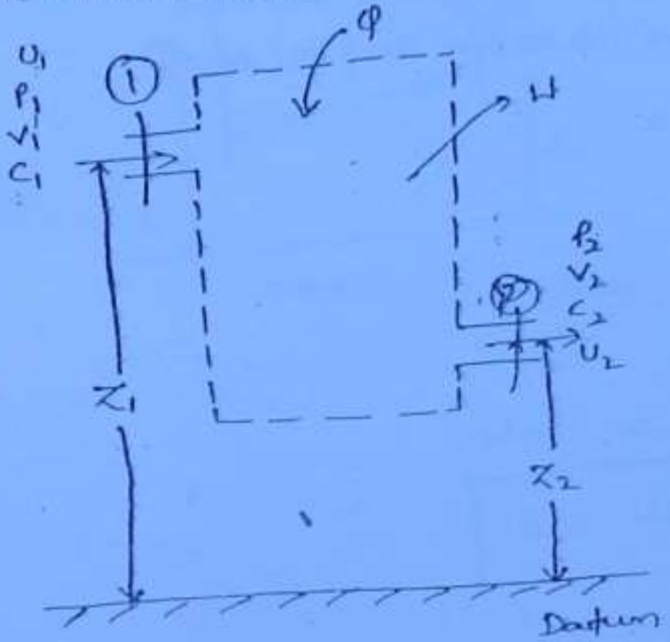
from mass conservation

$m_1 = m_2$

→*1 STEADY FLOW ENERGY EQUATION (SFEE) :-

A flow is said to be steady flow if properties do not vary with respect to time at any given section.

for steady flow, there is no accumulation of mass and energy in the control volume i.e. mass entering is equal to mass leaving and energy entering = energy leaving the control volume.



$W = W_{Entry\ flow} + W_{cv} + W_{Exit\ flow}$

$W = -P_1V_1 + W_{cv} + P_2V_2$

for steady flow, $E_1 = E_2$.

$$\therefore \frac{1}{2}mC_1^2 + mgZ_1 + U_1 + Q = \frac{1}{2}mC_2^2 + mgZ_2 + U_2 + W$$

$$\Rightarrow \frac{1}{2}mC_1^2 + mgZ_1 + U_1 + Q = \frac{1}{2}mC_2^2 + mgZ_2 + U_2 + (-P_1V_1) + W_{cv} + P_2V_2$$

$$\therefore \frac{1}{2}mC_1^2 + mgZ_1 + \underbrace{U_1 + P_1V_1} + Q = \frac{1}{2}mC_2^2 + mgZ_2 + \underbrace{U_2 + P_2V_2} + W_{cv}$$

$$\therefore H_1 + \frac{1}{2}mC_1^2 + mgZ_1 + Q = \frac{1}{2}mC_2^2 + H_2 + mgZ_2 + W_{cv}$$

· Dividing the above equation by 'm' :-

$$\boxed{h_1 + \frac{C_1^2}{2} + gZ_1 + q = h_2 + \frac{C_2^2}{2} + gZ_2 + w_{cv}}$$

→ 1st law of thermodynamics for steady flow open system.

Here, C_1 and C_2 are velocities wrt 1 and 2 respectively.

$$\frac{P_1V_1}{\cancel{RT_1}} = \frac{P_2V_2}{\cancel{RT_2}} \Rightarrow P_1V_1 = P_2V_2$$

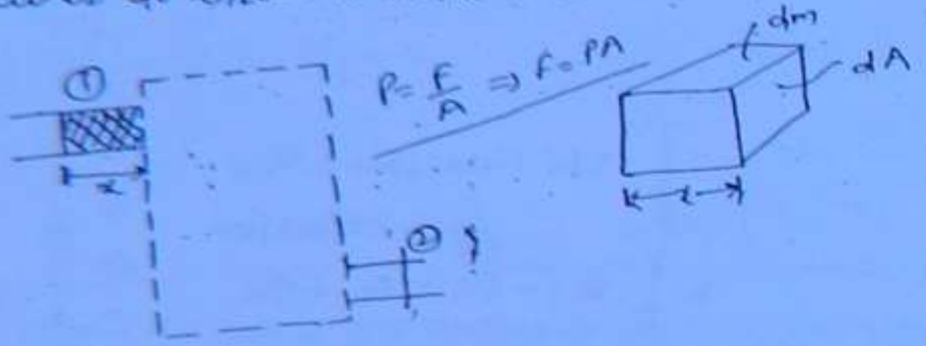
$$\Rightarrow 20 \times 15 = 1500 \times V_2$$

$$\therefore \boxed{V_2 = 0.2\ kgm} \quad \text{(Ans)}$$

5.4

—: FLOW WORK :—

It is the work transfer involved in causing the fluid element either to enter or leave the control volume is known as flow work.



$$\text{Work} = F \times x$$

$$= P\underbrace{dA \times x}$$

$$\text{Work} = PdV$$

$$dV = dA \times x.$$

$$v = \frac{dV}{dm}.$$

$$\text{Work} = PdV$$

$$\frac{\text{work}}{\text{mass}} = \frac{PdV}{dm}$$

$$\frac{\text{work}}{\text{mass}} = Pv$$

$$v = V/m$$

$$\text{Total Work} = P\underbrace{v.m}$$

$$\therefore \underline{V = v.m}$$

$$\boxed{\text{Total work} = PV.}$$

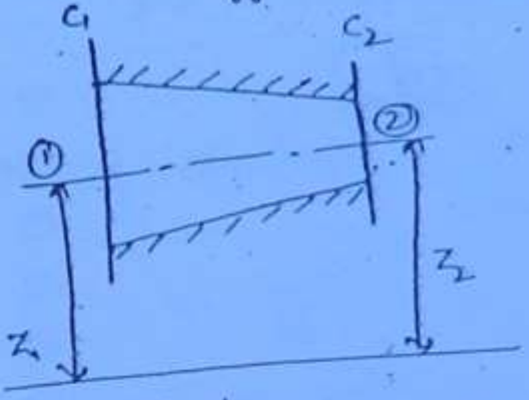
If the entry occurs at '1', then the entry flow work $= (-P_1V_1)$

If the exit occurs at '2', then the exit flow work $= (+P_2V_2)$.

This is the first law of thermodynamics equation for open system under steady flow conditions.

*** Special Cases :-

① **Nozzle :-** Nozzle is a device which is used for increasing kinetic energy at the expense of pressure energy.



Nozzle

$$h_1 + \frac{C_1^2}{2} + \cancel{Z_1 g} + \cancel{q}^{0} = h_2 + \frac{C_2^2}{2} + \cancel{Z_2 g} + \cancel{W_{cv}}$$

∵ Insulated, $q = 0$.

for a nozzle, $W_{cv} = 0$.

Also, $Z_1 = Z_2$ ∴ P.E are same

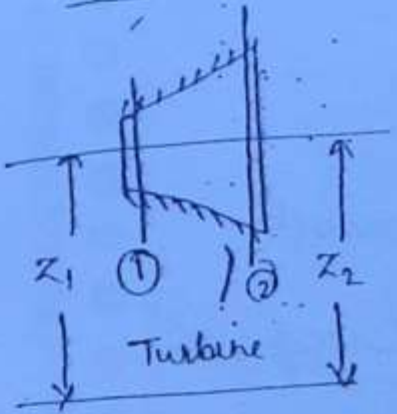
& Thus, $$\boxed{h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2}}$$

When $C_1 << C_2$, then

$$\boxed{h_1 = h_2 + \frac{C_2^2}{2}}$$

② **Turbine :-**

$$h_1 + \cancel{\frac{C_1^2}{2}} + \cancel{Z_1 g} + \cancel{q}^{0} = h_2 + \cancel{\frac{C_2^2}{2}} + \cancel{Z_2 g} \pm W_{cv}$$



Assumptions :-

① Steady flow

② Perfectly insulated, so ∴ $q = 0$.

③ Neglecting P.E changes.

④ Neglecting K.E changes.

$$h_1 = h_2 + W_{cv}$$

or, $$\boxed{W_{Turbine} = h_1 - h_2}$$

③ Compressor :-

Compressor work is always negative, Since work is done on the system.

$\boxed{W_{comp} = h_2 - h_1}$ (Assumptions are same as in case of turbine).

$w_{Turbine} = h_1 - h_2$

$w_{comp} = -w_{Turbine} = -(h_1 - h_2).$

$\therefore \boxed{w_{comp} = h_2 - h_1}$

④ Throttling valve :-

Examples of throttling :-

(i) flow through a partially opened valve.

(ii) flow through a very small opening (orifice).

(iii) flow through a porous plug.

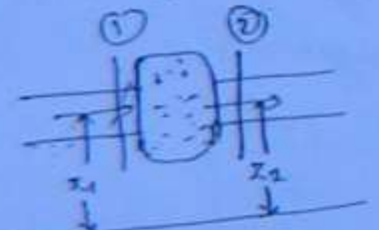
Characteristics of throttling :-

(i) No heat transfer.

(ii) No work transfer.

(iii) Highly irreversible process.

(iv) Enthalpy remains constant. (It is isenthalpic process).



$$h_1 + \frac{C_1^2}{2} + \cancel{Z_1 g} + \cancelto{0}{q} = h_2 + \frac{C_2^2}{2} + \cancel{Z_2 g} + \cancelto{0}{w_{cv}}$$

$$h_1 + \cancel{\frac{C_1^2}{2}} = h_2 + \cancel{\frac{C_2^2}{2}}$$

In comparision to enthalpy values, K.E changes are negligible.

$\therefore \boxed{h_1 \simeq h_2}$

Ideal Gas ($PV = mRT$)

Perfect Gas

↓

C_p & C_v are constants

(57)

Semi-Perfect Gas

↓

C_p & C_v ~~are~~ vary w.r.t tem.

No. of moles $(n) = \dfrac{\text{mass } (m)}{\text{Molecular wt. } (M)}$

$n = \dfrac{m}{M}$

or $m = nM$

∴ Ideal gas equation $PV = mRT$

$= n\underbrace{MR}T$

$MR = \bar{R}$ (universal gas constant).

$\therefore \boxed{PV = n\bar{R}T}$

$\bar{R} = 8.314$ kJ/kg K.

$R =$ characteristic gas constant.

Isothermal work $= mRT \ln \dfrac{V_2}{V_1}$

$= \left(n\bar{R}T \ln \dfrac{V_2}{V_1}\right).$

$$\boxed{R = \frac{\bar{R}}{M} = \frac{8.314}{M}}$$

Work Book   Chapter-2

① — ⓐ
② — ⓒ
③ — ⓐ
④ — ⓒ
⑤ — ⓒ
⑥ — ⓓ
⑦ — ⓐ
⑧ — ⓒ
⑨ — ⓐ
⑩ — ⓑ

⑪ — ⓑ
⑫ — ⓒ
⑬ — ⓒ
⑭ — ⓒ
⑮ — ⓑ
⑯ — ⓒ
⑰ — ⓒ
⑱ — ⓓ
⑲ — ⓓ
⑳ — ⓓ

㉑ — ⓑ
㉒ — ⓒ
㉓ — ⓒ
㉔ — ⓐ
㉕ —
㉖ — ⓑ
㉗ — ⓐ
㉘ — ⓒ
㉙ — ⓐ
㉚ — ⓑ

㉛ — ⓐ
㉜ — ⓐ
㉝ — ⓐ
㉞ — ⓐ
㉟ — ⓐ
→ (Assignment) 1153
㊱ — ⓐ
㊲ — ⓒ
㊳ — ⓐ
㊴ — ⓒ
㊵ — ⓐ
㊶ — ⓓ

① $T = C$, $n = 70$, $T = 354K$, $V_2 = 1 m^3$, $W = -206$ kj

(58)

$$W = n\bar{R}T \ln \frac{V_2}{V_1}$$

$$\Rightarrow -206 = 70 \times 8.314 \times 354 \ln\left(\frac{V_2}{V_1}\right)$$

$$\Rightarrow \ln\left(\frac{V_2}{V_1}\right) = \frac{-206}{70 \times 8.314 \times 354}$$

$$\text{or } \left(\frac{V_2}{V_1}\right) = e^{(9.9989845 \times 10^{-4})} = 1.001 \simeq 1.$$

$$\therefore \underline{\underline{\left(\frac{V_2}{V_1}\right) = 1}} \quad \therefore \underline{\underline{V_2 = V_1}}$$

③ ~~dQ_T = dW~~ $\underline{dQ_T = dW}$

$P_i = P_0$ and $T_i = T_0$

$T =$ Const (Isothermal Process).

$P_f = P_1$

$PV = ~~nRT~~ c$

$PV = C$

$P_i V_i = P_f V_f$

$$\underline{\frac{V_f}{V_i}} = \frac{P_i}{P_f}$$

$$W = Q = nRT \ln \frac{V_f}{V_i}$$

$$W = Q = nRT \ln \frac{P_i}{P_f}$$

$$Q = 1 \times R \times T_0 \ln\left(\frac{P_i}{P_f}\right)$$

$$Q = RT_0 \ln\left(\frac{P_0}{P_1}\right)$$

$$Q = -RT_0 \ln\left(\frac{P_1}{P_0}\right).$$

$-60\,\text{J/s} = \frac{}{}$

③ $W = -60\,W. = (60 \times 60) = 3600\,J$

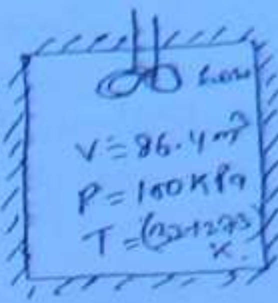
$V = 86.4\,m^3.$

$T_1 = 32°C, \quad T_2 = ?$

$C_v = 0.718, \quad R = 0.287\,kJ/kg\,K.$

$P_1 = 100\,kPa.$

$T_1 = (273+32)\,K = 305\,K.$



$$W = -60\,j/s = -\frac{60 \times 60 \times 60 \times 4}{1000}\,kj$$

$$= \underline{-864\,kj}$$

Air can be treated as an ideal gas.

$\therefore dU = mC_v dT.$

$dQ = dU + dW.$

$\because$ Insulated, $\therefore dQ = 0.$

$\therefore 0 = mC_v dT + (-864)$

$\Rightarrow mC_v dT = 864$

$\Rightarrow dT = \frac{864}{mC_v}$ — ①

Now for ideal gas $PV = mRT$

$\Rightarrow m = \frac{PV}{RT} = \frac{100 \times 86.4}{0.287 \times 305}$

$\therefore \underline{m = 98.7\,kg}$

$$\therefore dT = \frac{864}{98.7 \times 0.718} = 12.19 \simeq \underline{12°C.}$$

⑦ ~~$dQ = dU + dW$~~

~~$T_1 = 15°C, \quad T_2 = 45°C, \quad P = 100\,kPa.$~~

~~for Constant volume process, $V = C$~~

~~$dQ_v = dU = mC_v dT.$~~

~~$= 1 \times 3.5107 \times 30 = \underline{105.32\,kJ}$~~

$\frac{C_p}{C_v} = K$

$\Rightarrow C_v = \frac{C_p}{K} = \frac{1.042}{0.2968}$

$= \underline{3.570}$

④ Area of cycle = Area enclosed by the curve:-

Ist case Area of $\Delta = \frac{1}{2} bh$

$= \frac{1}{2} \times 8 \times 3 = \underline{12}$

2nd case, Area $= \frac{1}{2} \times 6 \times 4 = 12$

but for clockwise dir^n ~~area~~ work is ⊕ve
and for anticlock wise dir^n work is ⊖ve,

$\frac{C_p}{C_v} = R$

$(\because \frac{C_p}{R})$

⑧ ⑦ A-2, B-4, C-1, D-3.

⑧

⑦ $m = 1$ kg, $T_1 = 15°C$, $P = 100$ kPa, $T_2 = 45°C$.

$C_p = 1.042$ kJ/kg-K. $R = 0.2968$.

$Q_v = m c_v dT$

$= 1 \times 0.7452 \times 30$

$=$

$(C_p - C_v) = R$

$1.042 - 0.2968$

$= 0.7452$

8/2011

$(C_p - C_v) 6 = R.$

$(5.19 - C_v) = \frac{8.314}{4}$

$\Rightarrow C_v = (5.19 - 2.0785) = \underline{3.11}$

⑤ $\cancel{dQ}^{0} = dU + dW.$

$dW = \frac{10\cancel{0} \times 6\cancel{0} \times 3\cancel{0}}{1\cancel{000}} = \underline{-180 \text{ kJ}}$

$dW = (-180 \text{ kJ})$

$dU = m C_v dT$

$= 30 \times 0.7 \times dT$

$dU = -(dW)$

$\Rightarrow 30 \times 0.7 \times dT = +180$

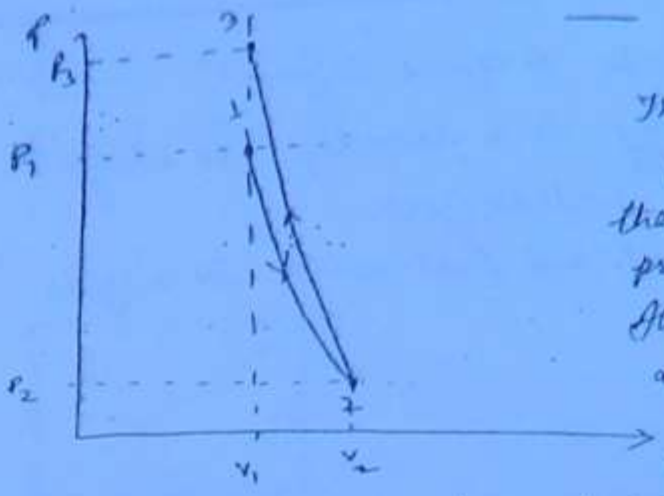
$\Rightarrow dT = \underline{\underline{+8.57}}$

$\therefore T_2 = T_1 + dT$

$= 21 + 8.57$

$= \underline{29.57°C}.$

⑯



The slope of adiabatic process is always greater than the slope of isothermal process.

Also, expansion work is ⊕ve and compression work is ⊖ve

∵ Compression work here is greater

∴ $W < 0$. (∵ W is ⊖ve)

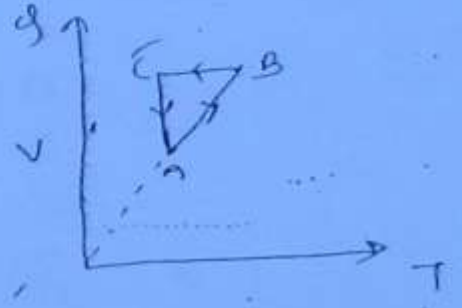
& $P_3 > P_1$.

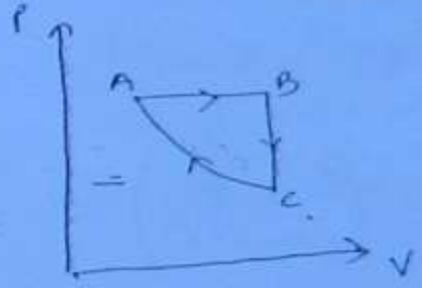
⑰ The total work done by the gas is $(300 \times 0.01) = 3$ kj

Out of this total work, $(100 \times 0.01) = 1$ kj is used in displacing atmospheric air.

Therefore the work that could be utilised $= (3-1) = \underline{2 \text{ kj}}$

⑲





$PV = mRT$

$V = \frac{mR}{P} \cdot T$

$y = M \cdot x$

$M = \frac{mR}{P} = \text{const}$

M is const if P is constant ∴ $AB \rightarrow \underline{P = C}$.

$BC \rightarrow V = C$

$CA \rightarrow T = C$

$AB \rightarrow P = C$.

⑳ For isothermal process of an ideal gas open system work is equal to closed system work because a rectangular hyperbola is symmetric about 'x' and 'y' axis and hence it gives same areas.

21) Change in property for any cycle is equal to zero.

Since internal energy is a property, so change in internal energy for a cycle will be zero.

As the ~~start~~ initial and final points in a cycle are same.

22) - (c)

23) for 300 W = $\frac{3\cancel{0}0 \times 6\cancel{0} \times 15}{1\cancel{000}}$ = 2700 kJ

69

$\cancel{d}P = 0$

(25) Assignment

$P_1 = 10$ bar, $V_1 = 1\,m^3$, $T_1 = 300\,K$.

$V_2 = 2\,m^3$.

~~Vdw~~ =

$\left(P + \frac{a}{V^2}\right)V = RT = C$

$\left(P_1 + \frac{a}{V_1^2}\right)V_1 = \left(P_2 + \frac{a}{V_2^2}\right)V_2$

$\Rightarrow \left(10 + \frac{a}{1}\right) \times 1 = \left(P_2 + \frac{a}{2^2}\right)2$

$= (10+a) = \frac{4P_2 + a}{2}$

$\Rightarrow 20 + 2a = 4P_2 + a$

$\Rightarrow a = 4P_2 - 20$

$\Rightarrow P_2 = \frac{a+20}{4} = \frac{a}{4} + 5$

$$\boxed{P_2 = \frac{a}{4} + 5}$$

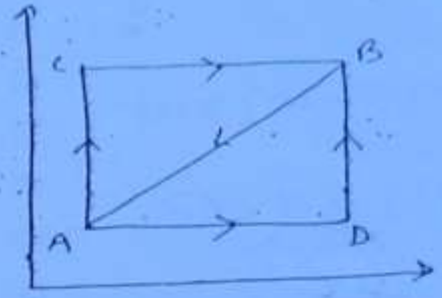
$\because a > 0$ and also its value is slightly greater than 0, $\therefore \frac{a}{4}$ will be very less.

$\therefore P_2$ will be slightly more than 5 bar.

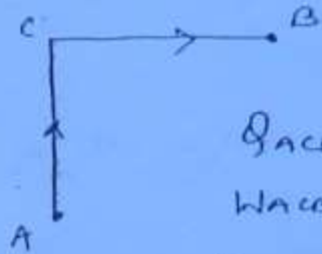
(27) E - G - I - K - N
F - H - J - K - M

Boundary and transient both are same and can be used for both heat and work

(28)



(63)



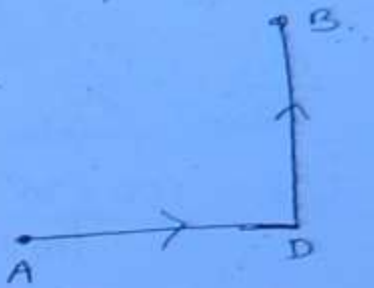
$Q_{ACB} = 180$

$W_{ACB} = 130$

$dQ_{ACB} = dU_{ACB} + dW_{ACB}$

$\Rightarrow 180 = (U_B - U_A) + 130$

$\therefore \boxed{(U_B - U_A) = 50}$ — ①



$Q_{ADB} = ?$

$W_{ADB} = 40\,kJ$

$dQ_{ADB} = dU_{ADB} + dW_{ADB}$

$dQ_{ADB} = 50 + 40$

$\therefore \boxed{dQ_{ADB} = 90\,kJ}$ (A)

(29) ~~$P_1V_1 = P_2V_2$~~

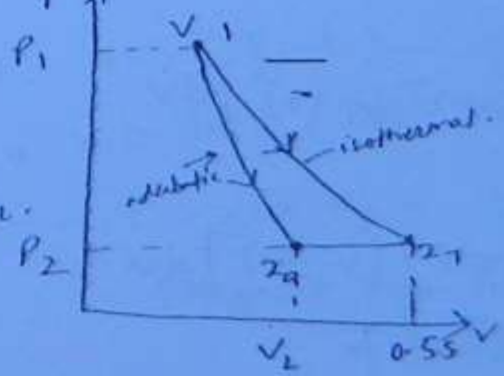
~~$P_1V_1 = \frac{P_1}{10} \times 0.55$~~

~~$V_1 = 0.055\,m^3$~~

for adiabatic ~~$P_1V_1^{\gamma} = P_2V_2^{\gamma}$~~

~~$\Rightarrow P_1 \times (0.055)^{1.4} = \frac{P_1}{10} \times (V_2)^{1.4}$~~

All values are greater than 0.55 but only value 0.40 is lesser.



(56)

$T_1 = 293\,K$
$P_1 = 100\,kPa$
$V_1 = 15\,m^3$

$T_2 = ?$
$P_2 = 150$
$V_2 = 25\,m^3$

$$PV = mRT$$

$$m = \frac{PV}{RT}$$

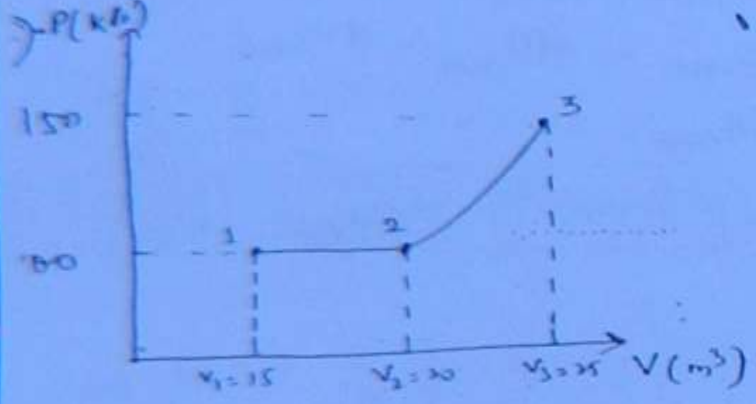
(64)

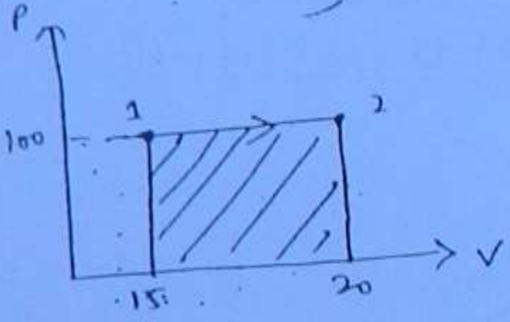
$$m_1 = m_2$$

$$\therefore \frac{P_1V_1}{\cancel{R}T_1} = \frac{P_2V_2}{\cancel{R}T_2}$$

$$\Rightarrow T_2 = \frac{150 \times 25 \times 293}{100 \times 15}$$

$$= 732.5 \approx \underline{\underline{733\,K}}$$





$$P = P_0 + 2(V - V_0)^2$$

$$W_{12} = P(V_2 - V_1)$$
$$= 100(20 - 15)$$
$$= \underline{\underline{500\,kJ}}$$

Again

$$W = \int P\,dV$$

$$W_{23} = \int_{V_2}^{V_3} \left[P_0 + 2(V - V_0)^2\right] dV$$

$$= \int_{V_2}^{V_3} \left[100 + 2(V - 20)^2\right] dV$$

$$W_{23} = \int_{20}^{25} 100\,dV + 2(V-20)^2\,dV$$

$\boxed{1\text{ bar} = 100\text{ kPa}}$

$$= \left[100V + \frac{2}{3}(V-20)^3 \right]_{20}^{25}$$

$$= 100 \times (25-20) + \frac{2}{3}\left[(25-20)^3 - (20-20)^3\right].$$

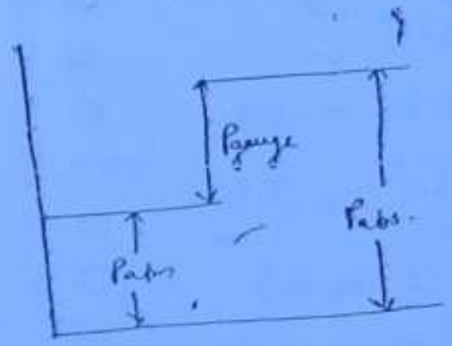
$$= 500 + \frac{2}{3}(5)^3 = 500 + 83 = 583\text{ kJ}.$$

65

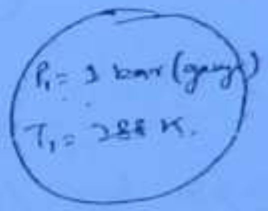
$$\therefore W_{Total} = W_{12} + W_{23}$$

$$= (500+583)\text{ kJ} = \underline{\underline{1083\text{ kJ}}}$$

(38) In the equation $PV = mRT$, 'P' is in absolute scale.



$P_{abs} = P_g + P_{atm}$



$P_{abs} = P_g + P_{atm}$

$P_1 = 1 + 1.013$

$P_1 = 2.013$ bar

$PV = mRT$ $\quad$ ($\because$ 1 bar = 100 kPa)

$$\Rightarrow m = \frac{PV}{RT} = \frac{2.013 \times 100 \times 2500}{0.287 \times 288 \times 10^{-6}}$$

$$\Rightarrow m = 6.08 \times 10^{-3}\text{ kg}.$$

For a constant volume process,

$$dQ_v = dU$$

$$dQ_v = mC_v\,dT = mC_v(T_2 - T_1)$$

$$= 6.08 \times 10^{-3} \times 0.718 \times (278 - 288)$$

$$\Rightarrow dQ_v = -43.7 \times 10^{-3}\text{ kJ} = -43.7\text{ joule}.$$

(Negative sign indicates rejection of heat).

$P_1 = 2.013$
$T_1 = 288$

$P_2 = ?$
$T_2 = 278$

$$P\overset{c}{\cancel{V}} = \overset{c}{\cancel{m}}\overset{c}{\cancel{R}}T$$

$$\frac{P_2}{P_1} = \frac{T_2}{T_1}$$

(66)

$$\Rightarrow P_2 = \frac{T_2 \times P_1}{T_1} = \frac{278 \times 2.013}{288}$$

$$\therefore P_2 = 1.943 \text{ bar.}$$

$$\therefore (P_2)_{gauge} = P_2 - P_{atm}$$
$$= 1.943 - 1.013$$
$$\therefore (P_2)_{gauge} = \underline{0.93 \text{ bar}} \text{ (Ans).}$$

5) $P_1 = ?$
$T_1 = 288$

$P_2 = 1 \text{ bar (gauge)}$
$T_2 = 278$

$$P_2 \text{ abs} = 1 + 1.013 = \underline{2.013 \text{ bar}}$$

$$\frac{P_1}{P_2} = \frac{T_1}{T_2}$$

$$\Rightarrow P_1 = \frac{2.013 \times 288}{278} = 2.085 \text{ bar}$$

$$(P_1)_{gauge} = (P_1)_{abs} - P_{atm}$$
$$= 2.085 - 1.013$$

$$\therefore (P_1)_{gauge} = \underline{1.07 \text{ bar}} \text{ (gauge).}$$

④ $W_{2-3} = P(V_3 - V_2)$

(40)

Process 1→2

$P_1V_1 = P_2V_2$

$\Rightarrow 1 \times 1.6 = P_2 \times 0.2$

$\Rightarrow P_2 = \frac{1.6}{0.2} = 8 \text{ bar}$

$W_{2-3} = 800\,(1.6 - 0.2) = 1120 \text{ kj}$

(41) Heat transfer for constant ~~process~~ pressure process.

~~$dQ_P = dH$~~

~~$= d(U+PV)$~~

$dQ_P = dU + dW$

$dQ_{2-3} = dU_{23} + \overset{1120}{dW_{23}}$

* Process 1-2 is an isothermal process.

∴ $U_1 = U_2$. (∵ Change in internal energy for isothermal process is 0).

$dU = U_3 - U_2$

$U_3 - U_2 = U_3 - U_1 = 3549 \text{ kj}$

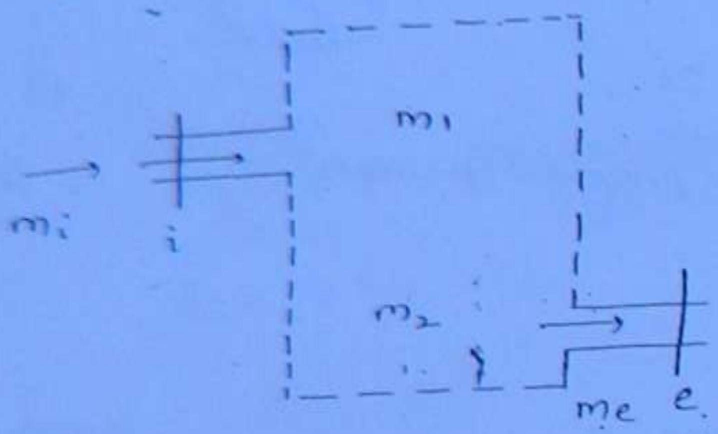
∴ $dQ_{23} = (3549 + 1120) \text{ kj}$

$= 4669 \text{ kj}$

← : UNSTEADY STATE FLOW : →

Let 'm$_i$' and 'm$_e$' be the masses entering and leaving the control volume.

Let 'm$_1$' and 'm$_2$' be the masses in the control volume initially and finally respectively.



m → kg

$$\dot{m} = \frac{m}{t} = \frac{kg}{sec}.$$

onservation of mass:-

$$\left(\frac{dm}{dt}\right)_{cv} = \frac{dm_i}{dt} - \frac{dm_e}{dt}$$

$$\boxed{\left(\frac{dm}{dt}\right)_{cv} = \dot{m}_i - \dot{m}_e} \quad ——— ①$$

onservation of Energy:-

nergy per unit mass $= h_i + \frac{C_i^2}{2} + z_i g + q.$

Total inlet energy, $E_i = m_i h_i + \frac{1}{2} m_i C_i^2 + m_i g z_i + Q.$

llary, Total energy leaving,

$$E_e = m_e h_e + \frac{1}{2} m_e C_e^2 + m_e g z_e + W_{cv}$$

Now,

$$\left(\frac{dE}{dt}\right)_{cv} = \frac{dE_i}{dt} - \frac{dE_e}{dt}$$

$$\left(\frac{dE}{dt}\right)_{cv} = \frac{d}{dt}\left(m_i h_i + \frac{1}{2} m_i c_i^2 + m_i g z_i + Q\right) - \frac{d}{dt}\left(m_e h_e + \frac{1}{2} m_e c_e^2 + m_e g z_e + W_{cv}\right).$$

Neglecting K.E & P.E changes, we have.

$$\left(\frac{dU}{dt}\right)_{cv} = \frac{d}{dt}\left(m_i h_i + Q\right) - \frac{d}{dt}\left(m_e h_e + W_{cv}\right)$$

$$\left(\frac{dU}{dt}\right)_{cv} = \frac{d}{dt} m_i h_i + \dot{Q} - \frac{d}{dt}(m_e h_e) + \dot{W}_{cv}$$

$\Big[\because$ $E = KE + PE + U$.
$dE = d(KE) + d(PE) + dU$
If $d(KE)$ & $d(PE)$ are neglected, then $dE = dU\Big]$

(69)

Let h_i & h_e are assumed to be constant with respect to time, then

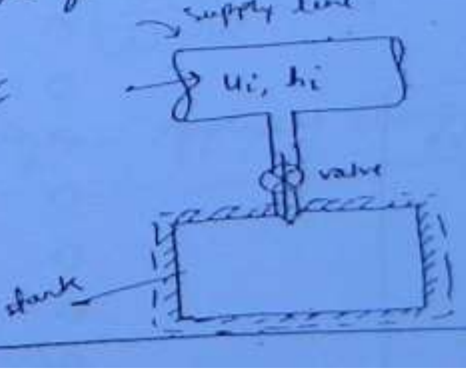
$$\left(\frac{dU}{dt}\right)_{cv} = h_i \frac{dm_i}{dt} + \dot{Q} - h_e \frac{dm_e}{dt} - \dot{W}_{cv}$$

$$\text{or, } \boxed{\left(\frac{dU}{dt}\right)_{cv} = \dot{m}_i h_i + \dot{Q} - \dot{m}_e h_e - \dot{W}_{cv}} \quad \text{—②}$$

PROBLEM

Q:- An insulated storage tank that is initially evacuated is connected to a supply pipeline carrying a fluid at specific internal energy 'u_i' and specific enthalpy 'h_i'. The valve is opened and fluid flows into the tank from the supply line and reaches the pressure same as that of supply pipe line. Show that the final specific internal energy of the fluid in the tank is equal to 'h_i'.

Soln:-



$m_1 = 0$ (evacuated)

Applying mass conservation :-

$$\left(\frac{dm}{dt}\right)_{cv} = \dot{m}_i - \dot{m}_e$$

$\dot{m}_e = 0$ (because no mass is leaving C.V)

$$\left(\frac{dm}{dt}\right)_{cv} = \dot{m}_i \quad \text{——} \quad ①$$

⑩

Now Energy conservation :

$$\left(\frac{dU}{dt}\right)_{cv} = \dot{m}_i h_i + \underset{(\text{insulated})}{\cancel{\dot{Q}}^{\,0}} - \underset{(\because \dot{m}_e = 0)}{\cancel{\dot{m}_e h_e}^{\,0}} - \underset{(\text{No work done})}{\cancel{\dot{W}}^{\,0}}.$$

$$\left(\frac{dU}{dt}\right)_{cv} = \dot{m}_i h_i \quad \text{——} \quad ②$$

$$\left(\frac{dU}{dt}\right)_{cv} = h_i \left(\frac{dm}{dt}\right)_{cv}.$$

Integrating , we have.

$$(dU)_{cv} = h_i \, (dm)_{cv}$$

$$U_2 - U_1 = h_i \, (m_2 - m_1)$$

$$m_2 u_2 - m_1 u_1 = h_i (m_2 - m_1)$$

$$\because m_1 = 0.$$

$$\therefore \cancel{m_2} u_2 = h_i (\cancel{m_2})$$

$$\therefore \boxed{u_2 = h_i} \text{ proved.}$$

Note :- If the fluid flowing is an ideal gas,

$$u_2 = h_i$$

$$C_v T_2 = C_p T_i$$

$$T_2 = \frac{c_p}{c_v} T_i$$

$$\therefore \boxed{T_2 = \gamma T_i}$$

Therefore 'T_2' will always be greater than T_i.

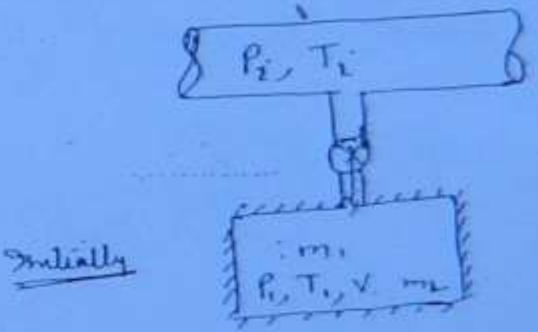
$$\left[\begin{array}{l} U = m c_v dT \\ u = c_v T \\ h = c_p T \end{array}\right]$$

× ×

ESE-2011

Q. The Pressure-cylinder of volume 'V' contains air at a pressure 'P_1' and temperature 'T_1'. It is to be filled from a supply pipeline maintained at a constant pressure 'P_i' and temperature 'T_i'. Show that the final temp^n of air in the cylinder after it has been charged to the pressure same as the supply line pressure is given by $T_2 = \dfrac{\gamma T_i}{1 + \frac{P_1}{P_i}\left(\frac{\gamma T_i}{T_1} - 1\right)}$. The tank is insulated.

Sol^n: Applying mass conservation:

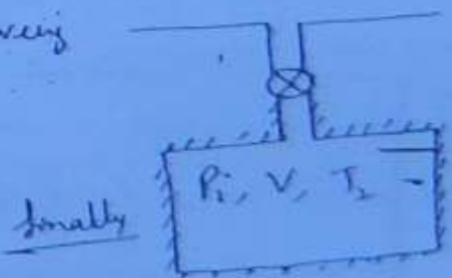


$$\left(\frac{dm}{dt}\right)_{cv} = \dot{m}_i - \dot{m}_e$$

$$PV = mRT$$

$$m = \frac{PV}{RT} \qquad \therefore \; m_2 = \frac{P_i V}{R T_2} \;\&\; m_1 = \frac{P_1 V}{R T_1}$$

$(\dot{m}_e) = 0$ (because no mass is leaving control volume).



$$\therefore \left(\frac{dm}{dt}\right)_{cv} = \dot{m}_i \quad \text{——} ①$$

Again, Applying Energy conservation:

(P.T.O)

Finally we get; ϕ

$U_2 - U_1 = (m_2 - m_1) h_i$ (as from last problem).

$m_2 u_2 - m_1 u_1 = (m_2 - m_1) h_i$

$\Rightarrow m_2 C_v T_2 - m_1 C_v T_1 = (m_2 - m_1) C_p T_i$ (72)

$\Rightarrow C_v (m_2 T_2 - m_1 T_1) = C_p T_i (m_2 - m_1)$.

$$\Rightarrow C_v \left(\frac{P_i V}{R} - \frac{P_1 V}{R} \right) = C_p T_i \left(\frac{P_i V}{R T_2} - \frac{P_1 V}{R T_1} \right)$$

$$\Rightarrow \frac{P_i V}{R} - \frac{P_1 V}{R} = \frac{C_p}{C_v} T_i \left(\frac{P_i V}{R T_2} - \frac{P_1 V}{R T_1} \right)$$

$$\Rightarrow (P_i - P_1) = \gamma T_i \left(\frac{P_i}{T_2} - \frac{P_1}{T_1} \right).$$

$$\Rightarrow (P_i - P_1) = \frac{\gamma T_i P_i}{T_2} - \frac{\gamma T_i P_1}{T_1}$$

$$\Rightarrow \cancel{P_i + P} \quad P_i - P + \frac{\gamma T_i P_1}{T_1} = \frac{\gamma T_i P_i}{T_2}$$

$$\Rightarrow T_2 = \frac{\gamma T_i P_i}{P_i - P_1 + \frac{\gamma T_i P_1}{T_1}}$$

$$T_2 = \frac{\gamma T_i P_i}{P_i \left[1 - \frac{P_1}{P_i} + \frac{\gamma T_i P_1}{P_i T_1} \right]}$$

$$\therefore \boxed{T_2 = \frac{\gamma T_i}{1 + \frac{P_1}{P_i} \left(\frac{\gamma T_i}{T_1} - 1 \right)}}$$

(Proved)

← : SECOND LAW OF THERMODYNAMICS : →

(Directional Law) ↦ (Concept of Entropy)

first law of thermodynamics simply says that energy is conserved. It does not give any direction for a particular process. It is the second law of thermodynamics which gives direction for a particular process through the concept of entropy - and hence second law is known as directional law.

Work is known as high grade energy. Heat is known as low grade energy.

It is found that, for a cycle, complete conversion of low grade energy (heat) into high grade E energy (work) is impossible.

** Thermal Energy Reservoirs (TER) :-

Source :- Source is a ~~rest~~ reservoir which supplies thermal energy without undergoing any temperature change.

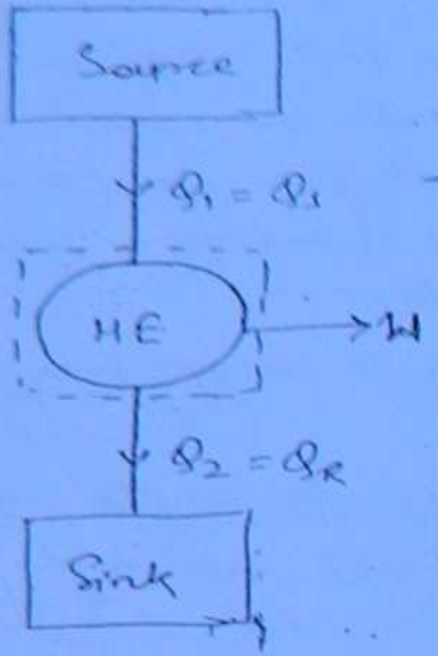
Sink : It is a reservoir which absorbs thermal energy without undergoing any temperature change.

** Statements of 2nd law of Thermodynamics :-

Ist ➤ KELVIN - PLANK STATEMENT :-

It is impossible to develop a device operating on a cycle which produces work while exchanging heat with a single reservoir. This device is known as PMM-II and the efficiency of PMM-II is 100% and therefore 100% efficiency is impossible from second law of thermodynamics.

* CONCEPT OF HEAT ENGINE :-



$$Q_1 = \cancel{Q}\ W + Q_2$$

$$W = Q_1 - Q_2$$

$$\eta = \frac{O}{I} = \frac{W}{Q_1}$$ (74)

$$\eta = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

$$\therefore \boxed{\eta = 1 - \frac{Q_R}{Q_S}} \quad \text{—} \ ①$$

Heat Engine is a device which converts part of heat into work and rejects remaining to sink or surrounding.

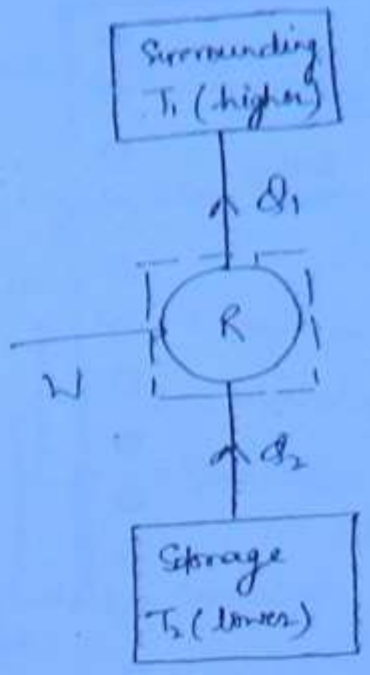
Equation ① is valid both for reversible and irreversible cycle.

2:- CLAUSIUS STATEMENT:-

It is impossible to transfer heat from lower temperature to the higher temperature without any external input.

* REFRIGERATOR :-

A refrigerator is a device which maintains lower temperature compared to surroundings. As lower temperatures are to be maintained continuously, refrigerator must operate on a cycle.



COP = $\frac{\text{Desired Effect}}{\text{Energy Input}}$

$$(COP)_R = \frac{Q_2}{W}$$

$$\boxed{(COP)_R = \frac{Q_2}{Q_1 - Q_2}} \quad \text{—②}$$

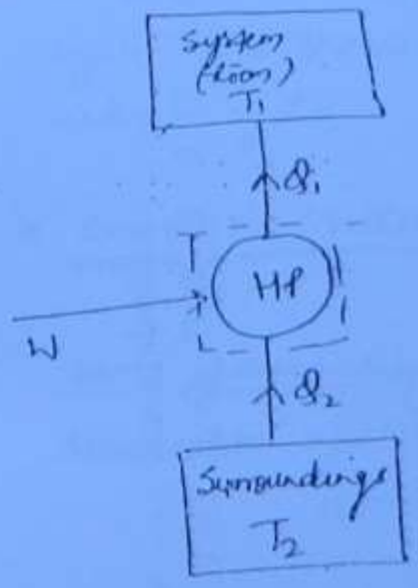
TS

$\because W + Q_2 = Q_1$

$\therefore \underline{W = Q_1 - Q_2}$

Equation ② is valid for reversible and irreversible refrigerators.

* HEAT PUMP :- Heat pump is a device which maintains higher temperature compared to surrounding.



$$(C.O.P)_{HP} = \frac{Q_1}{W}$$

$W + Q_2 = Q_1$

$W = Q_1 - Q_2$.

$$\boxed{(COP) = \frac{Q_1}{Q_1 - Q_2}} \quad \text{—③}$$

This equation is valid for reversible as well as irreversible heat pump.

**

N.B

RELATIONSHIP BETWEEN COP OF A HEAT PUMP AND COP OF A REFRIGERATOR OPERATING BETWEEN SAME TEMPERATURES :-

$$(COP)_{hP} = \frac{Q_1}{Q_1 - Q_2}$$

㉖

$$(COP)_R = \frac{Q_2}{Q_1 - Q_2}$$

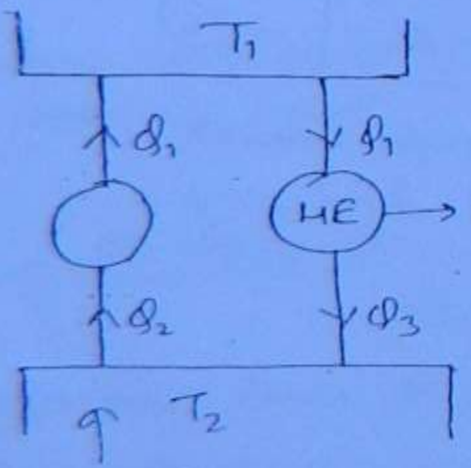
$$(COP)_{hP} - (COP)_R = \frac{Q_1}{Q_1 - Q_2} - \frac{Q_2}{Q_1 - Q_2}$$

$$(COP)_{hP} - (COP)_R = 1.$$

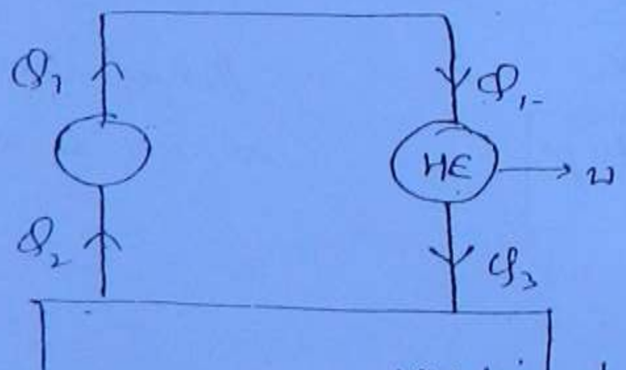
$$\text{or, } \boxed{(COP)_{hP} = 1 + (COP)_R.} \quad \text{——— (4)}$$

* TO SHOW THAT ~~A~~ KELVIN - PLANK AND CLAUSIS STATEMENTS ARE PARALLEL STATEMENTS OF 2ND LAW OF THERMODYNAMICS :-

ASE-1 : Violation of Clausius Statement :-



Violation of Clausius Statement



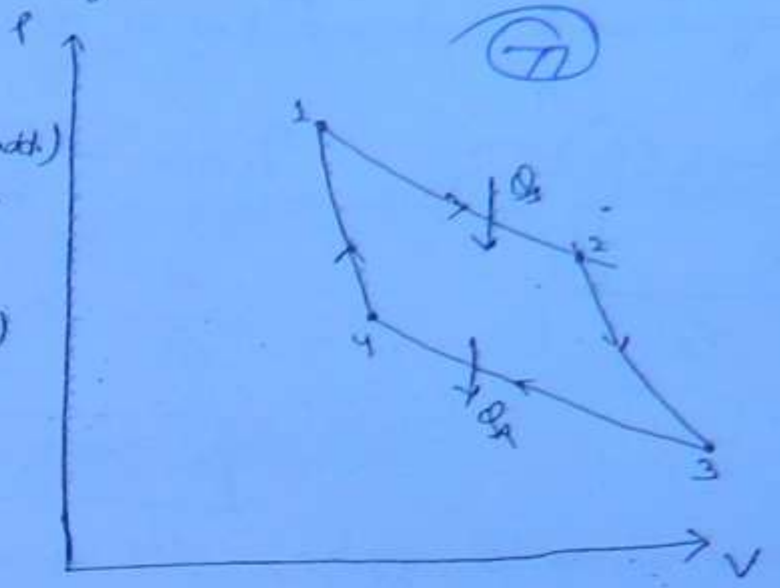
Violation of Kelvin - Plank Statement.

⇒* CARNOT CYCLE :- (Reversible Cycle) :-

A cycle is said to be a reversible cycle when each process in a cycle is reversible.

slow
- 1-2 → Isothermal expansion (heat add.)
- 2→3 → Adiabatic expansion
- 3-4 → Isothermal compression (heat rejection)
- 4-1 → Adiabatic compression.

fast



Isothermal → Slow
Adiabatic → fast.

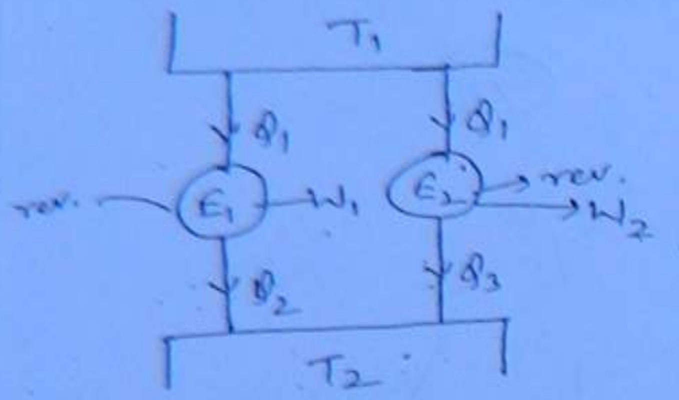
Carnot cycle consists of two isothermal processes and two adiabatic processes. Isothermal process is a slow process and adiabatic process is a fast process and hence, these two combinations in a cycle are not possible, therefore Carnot's cycle is a theoritical cycle and it is used for comparing other actual cycles.

* CARNOT'S THEOREM :-

for various cycles operating between same temperature limits, none has efficiency greater than reversible cycle efficiency

(P.T.O)

Let us consider two engines 'E₁' and 'E₂'. Let 'E₁' be irreversible engine and 'E₂' be reversible engine.



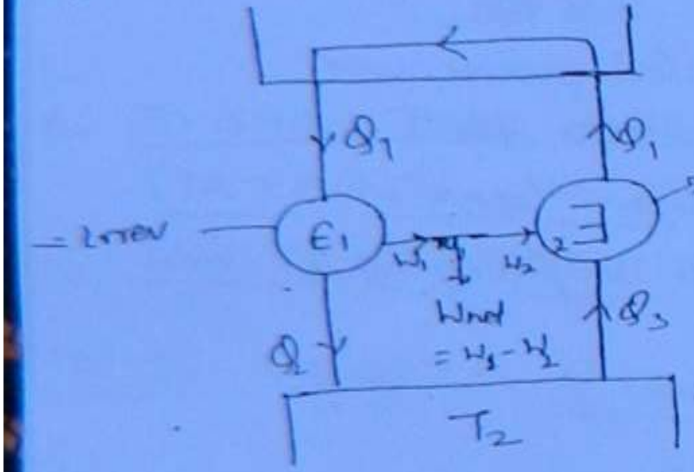
* Let us assume, $\eta_{irrev} > \eta_{rev}$.

$$\eta_{irrev} = \frac{W}{Q_1} \; ; \; \eta_{rev} = \frac{W_2}{Q_1}.$$

$$\frac{W_1}{\cancel{Q_1}} > \frac{W_2}{\cancel{Q_1}}$$

$$\therefore \boxed{W_1 > W_2}$$

As 'E₂' is a reversible engine, let us reverse the engine.



-- Violation of K-P statement.

$(\eta_{irrev} > \eta_{rev} \rightarrow \text{wrong})$

is is the violation of Kelvin-Plank statement and hence assumption is wrong. Similarly if we take the ficiency of both the engines to be same, then it would d to violation of Clausius statement. Therefore this is possible. And hence, efficiency of a reversible le is always greater than efficiency of irreversible le operating between same temperature limits.

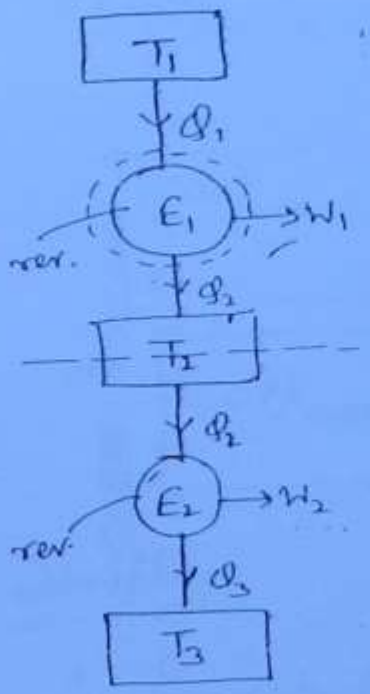
* Important Points with respect to reversible cycle:-

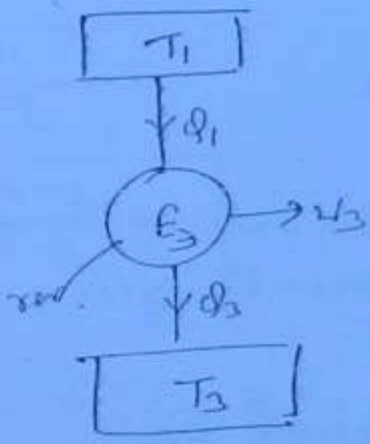
① Efficiency of all reversible cycles is equal if they operate between same temperature limits.

② Efficiency of a reversible cycle is independent of working fluid.

*** ③ <u>Efficiency of a reversible cycle depends only on temperature limits.</u>

⇒* THERMODYNAMIC TEMPERATURE SCALE:- (79)





$$\eta = 1 - \frac{Q_R}{Q_S}$$

$$\eta_1 = 1 - \frac{Q_2}{Q_1}$$

$$\eta_1 = f(T_1, T_2)$$

$$1 - \frac{Q_2}{Q_1} = f_1(T_1, T_2)$$

$$1 - f_1(T_1, T_2) = Q_2/Q_1$$

$$\frac{Q_1}{Q_2} = \frac{1}{1 - f_1(T_1, T_2)}$$

$$Q_1/Q_2 = \phi_1(T_1, T_2)$$

Similarly :-

$$\frac{Q_2}{Q_3} = \phi(T_2, T_3)$$

and $\frac{Q_1}{Q_3} = \phi(T_1, T_3)$.

$$\frac{Q_1}{Q_2} = \frac{\frac{Q_1}{Q_3}}{\frac{Q_2}{Q_3}} \quad \text{or} \quad \frac{Q_1}{Q_2} = \frac{\phi_3(T_1, T_3)}{\phi_2(T_2, T_3)}.$$

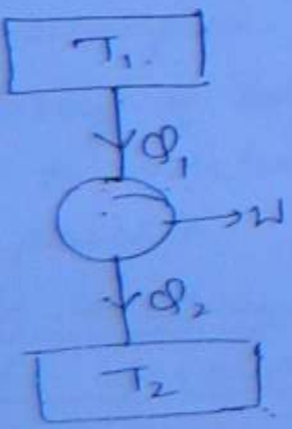
$$\frac{Q_1}{Q_2} = \frac{\psi_1(T_1)}{\psi_2(T_2)}$$

Based on kelvin's experiments.

$$\boxed{\frac{Q_1}{Q_2} = \frac{T_1}{T_2}}$$ → This is only valid for reversible cycle.

$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$) is valid for a reversible cycle.

* EFFICIENCY OF A REVERSIBLE ENGINE :-



$$\eta = 1 - \frac{Q_2}{Q_1}$$

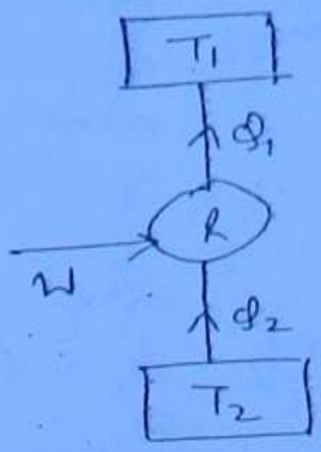
If the cycle is reversible,

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}.$$

$$\boxed{\eta_{rev} = 1 - \frac{T_2}{T_1}}$$

$$\boxed{\eta_{rev} = \eta_{max} = 1 - \frac{T_L}{T_H}}$$

⇒ * COP OF A REVERSIBLE REFRIGERATOR :-



$$COP_R = \frac{Q_2}{Q_1 - Q_2}$$

$$COP_R = \frac{Q_2}{Q_2\left[\frac{Q_1}{Q_2} - 1\right]}$$

$$(COP)_R = \frac{1}{\frac{Q_1}{Q_2} - 1}$$

for reversible refrigerator,

$$(COP)_{rev,R} = \frac{1}{\frac{T_1}{T_2} - 1}$$

$$\boxed{(COP)_{rev,R} = \frac{T_2}{T_1 - T_2}}$$

or $$\boxed{(COP)_{max,R} = (COP)_{rev,R} = \frac{T_L}{T_H - T_L}}$$

Similarly.

for reversible heat pump :-

~~(COP) HP =~~

$$\boxed{(COP)_{rev,HP} = \frac{T_H}{T_H - T_L}}$$

→ * CLAUSIUS INEQUALITY:-

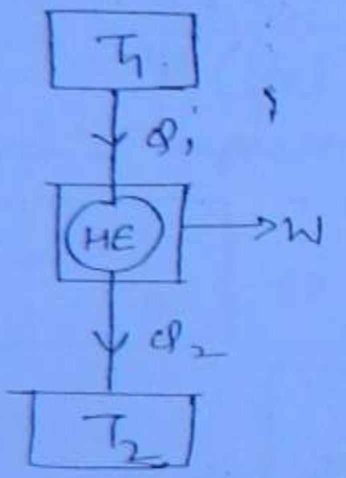
The cyclic integral of $\frac{dQ}{T}$ is less than or equal to zero.

$$\oint \frac{dQ}{T} \leq 0$$

$\oint \frac{dQ}{T} = 0 \Rightarrow$ cycle is reversible

$\oint \frac{dQ}{T} < 0 \Rightarrow$ cycle is irreversible.

se I:- Reversible Cycle:-



$$\oint_{rev} \frac{dQ}{T} = \frac{Q_1}{T_1} + \left(-\frac{Q_2}{T_2}\right) \quad ①$$

we know that for a reversible process,

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

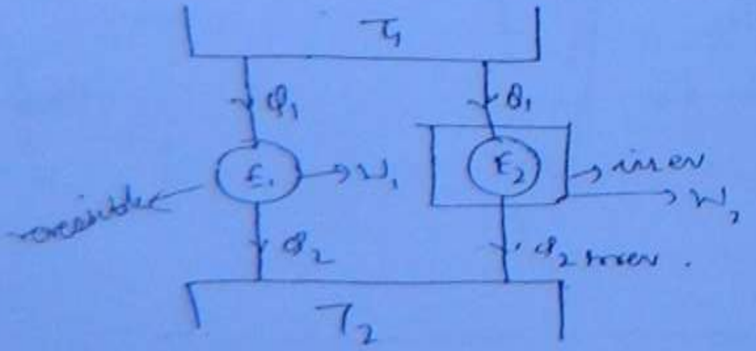
or. $$\frac{Q_1}{T_1} = \frac{Q_2}{T_2} \quad ②$$

substituting in eq? ①, we have

$$\therefore \oint \frac{dQ}{T} = \frac{Q_1}{T_1} - \frac{Q_1}{T_1}$$

$$\Rightarrow \boxed{\oint \frac{dQ}{T} = 0}$$

-II: Irreversible Cycle:-



$\eta_1 = \frac{W_1}{Q_1}$; $\eta_2 = W_2/Q_1$

$\eta_1 > \eta_2$

$$\frac{W_1}{\cancel{Q_1}} > \frac{W_2}{\cancel{Q_1}}$$

$\Rightarrow W_1 > W_2$.

for reversible process,

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}.$$

$$\oint_{\text{irrev.}} \frac{dQ}{T} = \oint \frac{Q_1}{T_1} + \left(-\frac{Q_2\,\text{irrev.}}{T_2}\right)$$

$$\oint_{\text{irrev.}} \frac{dQ}{T} = \frac{Q_1}{T_1} - \frac{Q_2\,\text{irrev}}{T_2}$$

$$\oint_{\text{irrev.}} \frac{dQ}{T} = \frac{Q_2}{T_2} - \frac{Q_2\,\text{irrev}}{T_2}$$

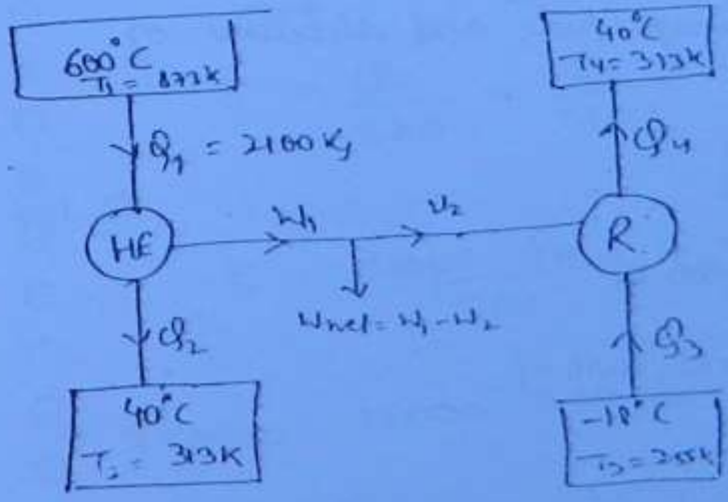
83

$$\oint_{\text{irrev.}} \frac{dQ}{T} = \frac{Q_2 - Q_2\,\text{irrev.}}{T_2}$$

$$\boxed{\oint \frac{dQ}{T} < 0 \; ; \; (Q_2 < Q_2\,\text{irrev.})}$$

PROBLEM

Q:- A reversible heat engine operates between 600°C and 40°C. ~~..~~ This engine drives a reversible refrigerator operating between 40°C and -18°C. Still there is a net work output of 370kj where the heat received by the engine is 2100 kjoule. Determine the cooling effect.



$$(COP)_R = \frac{T_L}{T_H - T_L}$$

$$\cancel{\frac{Q_3}{W_2}}$$

$$(COP)_e = \frac{Q_3}{W_2} = \frac{255}{313 - 255}$$

$$\therefore \frac{Q_3}{W_2} = 4.396$$

$W_1 = Q_1 - Q_2$

Also ∵ It is reversible cycle.

$$\therefore \frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$

$$\Rightarrow Q_2 = \frac{Q_1 \times T_2}{T_1} = \frac{2100 \times 313}{873}$$

$$\therefore Q_2 = \underline{752.9 \text{ kj}}$$

$$\therefore W_1 = (2100 - 752.9) = 1347.1 \text{ kj}$$

$$W_{net} = W_1 - W_2 = 370$$

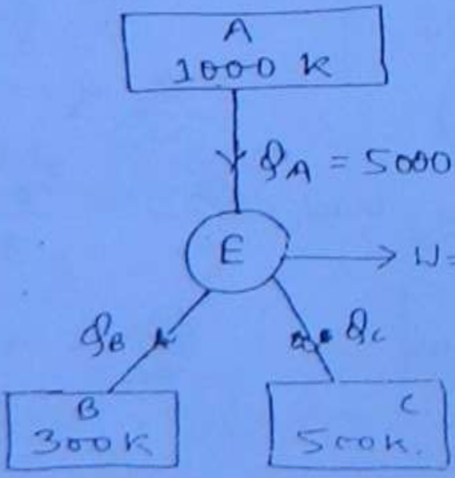
$$\therefore W_2 = \underline{977.1 \text{ kj}}$$

$$\therefore \frac{Q_3}{977.1} = 4.396$$

$$\therefore \boxed{Q_3 \text{ (Cooling Effect)} = 4295.2 \text{ kj}}$$ (Ans)

***

Figure shows a reversible cycle during which, it exchanges heat with three thermal reservoirs and develops 1000 kj of work. find the magnitude and direction of Q_B and Q_C.



~~let Q_B and Q_C are the heat rejected by the system~~

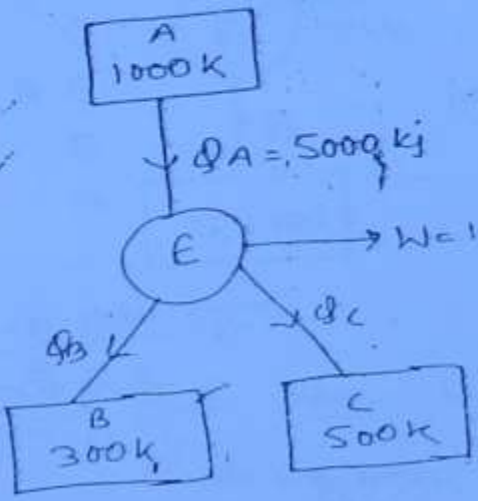
$\cancel{W = Q_A - (Q_B + Q_C)}$ — ①

$\cancel{\frac{Q_A}{T_A} = \frac{Q_B}{T_B}}$

$\cancel{\Rightarrow \frac{5000}{1000} = \frac{Q_B}{300} \Rightarrow Q_B = 1500 \text{ kj}}$

$\cancel{\frac{Q_A}{T_A} = \frac{Q_C}{T_C} \Rightarrow Q_C = \frac{5000}{1000} \times 500 = 2500 \text{ kj}}$.

let the direction of Q_B and Q_C are downwards.



Using Energy conservation:-

$Q_A = W + Q_B + Q_C$

$\Rightarrow 5000 = 1000 + Q_B + Q_C$

$\therefore Q_B + Q_C = 4000$ — ①

Again, using clausius inequality.

$$\oint_{rev} \frac{dQ}{T} = 0.$$

$$\frac{5000}{1000} + \left(-\frac{Q_C}{500}\right) + \left(-\frac{Q_B}{300}\right) = 0$$

$$5 = \frac{Q_C}{500} + \frac{Q_B}{300} \quad \text{— ②}$$

from eq^n ①,

$$5 = \frac{4000 - Q_B}{500} + \frac{Q_B}{300}$$

$$7500 = 12000 - 300 Q_B + 500 Q_B$$

$$\Rightarrow 200 Q_B = -4500$$

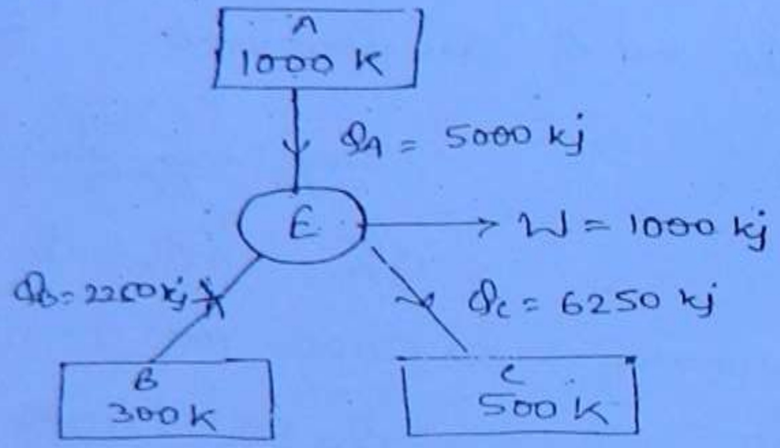
$$\therefore Q_B = \underline{-2250 \text{ kj}}$$

$$Q_B + Q_C = 4000$$

$$\Rightarrow -2250 + Q_C = 4000$$

$$\therefore Q_C = \underline{6250 \text{ kj}}$$

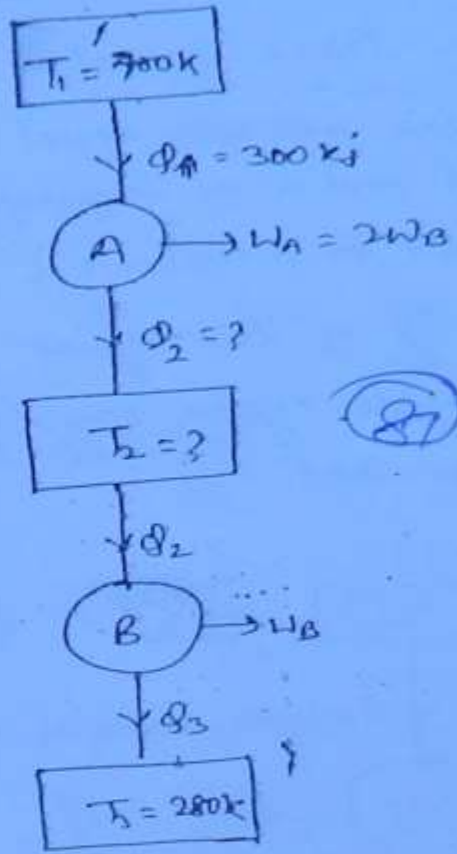
∵ The value of 'Q_B' is negative, it means that the direction assumed by us is wrong, hence the direction of 'Q_B' is upward.



Two reversible heat engines 'A' and 'B' are arranged in series. Heat Engine 'A' rejects heat directly to 'B'. Engine 'A' recieves 300 kj of heat at a temperature of 427°C from a high temperature source while engine 'B' rejects heat to a sink at 7°C. If the work output of 'A' is 2 times that of 'B', find :-

Intermediate temperature of 'A' and 'B'.
Efficiencies of each engine
Heat rejected by Engine 'A' i.e. heat recieved by Engine 'B'.
Heat rejected to sink.

Solⁿ:



(87)

~~$W_B = Q_2 - Q_3$~~

~~$2W_B = Q_1 - Q_2$~~

~~$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$~~

~~$W_A = 2Q_2 - 2Q_3$~~

~~$2Q_2 - 2Q_3 = Q_1 - Q_2$~~

~~$\Rightarrow 3Q_2 = 300 + 2Q_3$~~

~~$Q_2 = 100 + \frac{2Q_3}{3}$~~

~~$\frac{Q_2}{T_2} = \frac{Q_3}{T_3}$~~

~~$= \frac{100 + \frac{2Q_3}{3}}{} = \frac{Q_3}{T_3}$~~

~~$= 300 + 2Q_3 = \frac{3Q_3}{T_3}$~~

~~$= 300 \times 280 +$~~

for a reversible engine,

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2} \quad \text{—①} \qquad \text{Also} \qquad \frac{Q_2}{T_2} = \frac{Q_3}{T_3} \quad \text{—②}$$

from ① & ②

$$\frac{Q_1}{T_1} = \frac{Q_3}{T_3} \Rightarrow Q_3 = \frac{300 \times 280}{700} = 120 \text{ kJ}$$

$$W_A = 2W_B$$

$$\therefore Q_1 - Q_2 = 2Q_2 - 2Q_3 \Rightarrow 3Q_2 = Q_1 + 2Q_3$$

$$\Rightarrow Q_2 = \frac{300 + 240}{3} = 180 \text{ kJ}$$

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$

$$\Rightarrow T_2 = \frac{Q_2 \times T_1}{Q_1} = \frac{180 \times 700}{300} = 420 \text{ K.}$$

$$\eta_A = 1 - \frac{T_2}{T_1} = 0.4 = 40\%.$$

$$\eta_B = 1 - \frac{T_3}{T_2} = 0.33 = 33.3\%$$

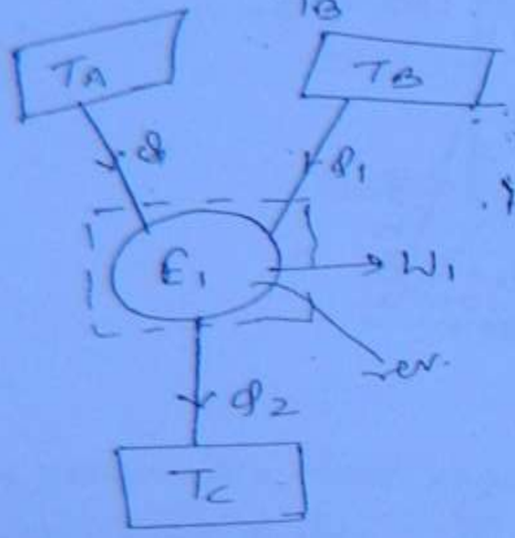
*****
Q.
V.V.I

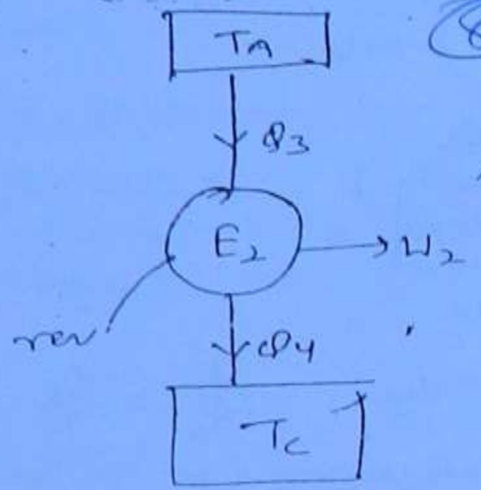
A reversible engine works between three thermal reservoirs 'A', 'B' and 'C'. The Engine receives equal amount of heat from reservoirs 'A' and 'B' at temperatures 'T_A' and 'T_B' respectively. and rejects heat to a reservoir 'C' at a temperature 'T_C'. If the efficiency of this engine is 'α' times the efficiency of a reversible engine operating between two reservoirs 'A' and 'C', Show that

$$\frac{T_A}{T_B} = 2(1-\alpha)\frac{T_A}{T_C} + (2\alpha - 1)$$

(88)

Solⁿ





$$\eta_1 = 1 - \frac{Q_2}{2Q_1} \quad \text{—} ①$$

$$\oint \frac{dQ}{T} = 0 \quad (\text{for rev. cycle})$$

$$\therefore \frac{Q_1}{T_A} + \frac{Q_1}{T_B} - \frac{Q_2}{T_C} = 0$$

$$Q_1\left[\frac{1}{T_A} + \frac{1}{T_B}\right] = \frac{Q_2}{T_C}$$

$$\frac{Q_2}{Q_1} = \frac{T_C}{T_A} + \frac{T_C}{T_B}$$

$$\eta_1 = 1 - \frac{1}{2}\left(\frac{T_C}{T_A} + \frac{T_C}{T_B}\right) \quad \text{———} ②$$

Now

$$\eta_2 = 1 - \frac{Q_4}{Q_3} \quad \text{——} ③$$

Again $\oint \frac{dQ}{T} = 0$.

$$\frac{Q_3}{T_A} - \frac{Q_4}{T_C} = 0 \Rightarrow \frac{Q_3}{T_A} = \frac{Q_4}{T_C}$$

or $\frac{Q_4}{Q_3} = \frac{T_C}{T_A}$

$$\therefore \eta_2 = 1 - \frac{T_C}{T_A} \quad \text{——} ④$$

(89)

But it is given that,

$$\eta_1 = \alpha \eta_2$$

$$\therefore 1 - \frac{1}{2}\left(\frac{T_C}{T_A} + \frac{T_C}{T_B}\right) = \alpha\left(1 - \frac{T_C}{T_A}\right) \quad \text{——} ⑤$$

Multiplying both sides with T_A.

∴ Eq= ⑤ becomes;

$$T_A - \frac{T_A}{2}\left(\frac{T_C}{T_A} + \frac{T_C}{T_B}\right) = \alpha T_A\left(1 - \frac{T_C}{T_A}\right)$$

$$\Rightarrow T_A - \frac{T_C}{2} - \frac{T_C}{2} \cdot \frac{T_A}{T_B} = \alpha T_A - \alpha T_C$$

$$\Rightarrow T_A - \frac{T_C}{2} - \alpha T_A + \alpha T_C = \frac{T_C}{2} \cdot \frac{T_A}{T_B}$$

$$\Rightarrow \frac{2}{T_C}\left[T_A - \frac{T_C}{2} - \alpha T_A + \alpha T_C\right] = \frac{T_A}{T_B}$$

$$\Rightarrow \frac{2T_A}{T_C} - 1 - 2\alpha\frac{T_A}{T_C} + 2\alpha = \frac{T_A}{T_B}$$

or, $\boxed{\frac{T_A}{T_B} = \frac{2T_A}{T_C}(1-\alpha) + 2\alpha - 1}$

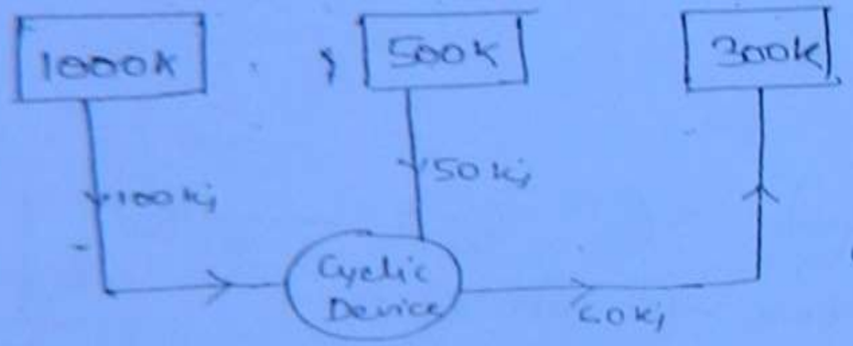
Work Book

Chapter - 4

① — ⓐ
② — ⓑ
③ — ⓒ
④ — ⓐ
⑤
⑥ — ⓓ
⑦ — ⓓ
⑧ — ⓑ
⑨ — ⓑ
⑩ — ⓑ
⑪
⑫
⑬
⑭
⑮
⑯
⑰
⑱
⑲

(96)

①



∵ Heat supplied > Heat Rejected.

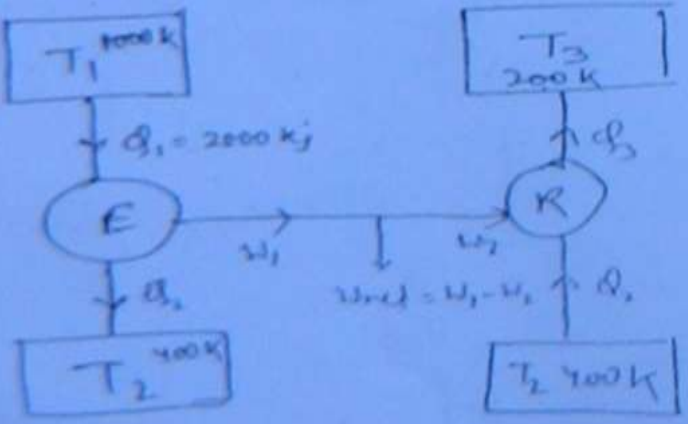
∴ There will work output. & If there is work output, it is an engine.

2:

Also, $\oint \frac{dQ}{T} = 0$

∴ It is reversible.

③



$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$

$\Rightarrow Q_2 = \frac{Q_1 \times T_2}{T_1} = \frac{2000 \times 400}{1000}$

$= 800$ kj

$W_1 = Q_1 - Q_2 = (2000 - 800) = 1200$ kj

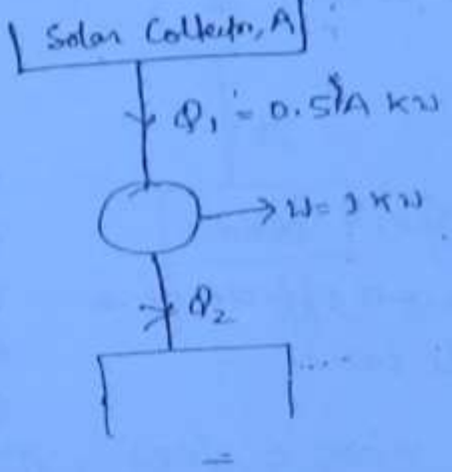
$W_2 =$ ~~Wnet~~ $W_1 - W_{net} = (1200 - 300) = 900$ kj

$W_2 = Q_2 - Q_3$

$\Rightarrow Q_3 = Q_2 - W_2 = 80$

(97)

③



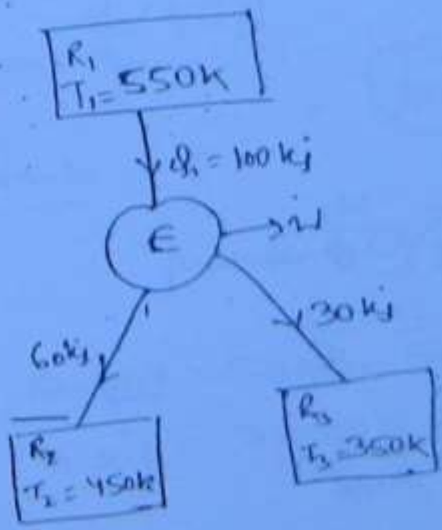
0.5 kW/m^2 $\times$ A m^2

$\eta = W/Q_1$

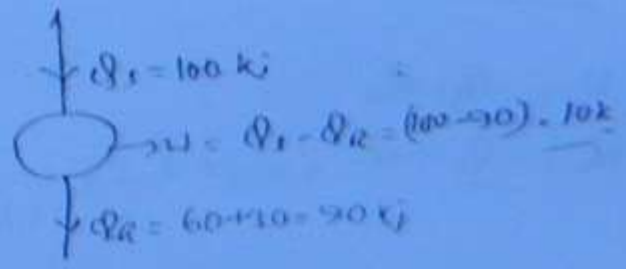
$\Rightarrow 0.2 = \frac{1}{0.5A}$

$\Rightarrow \boxed{A = 10}$

④



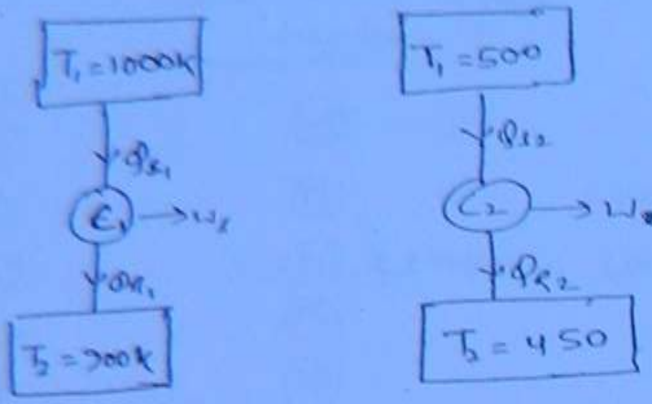
$\eta = \frac{\text{Output}}{\text{Input}}$



$\eta = \frac{10}{100} = 0.1 = 10\%$

ⓢ [illegible]

⑥



$W_1 = W_2 = W.$

$\eta_1 = 1 - \frac{T_2}{T_1} = 1 - \frac{900}{1000} = 0.1$

$\eta_1 = \frac{O}{I} = \frac{W}{Q_{S_1}}$

$\eta_2 = 1 - \frac{\overset{9}{\cancel{450}}}{\underset{10}{\cancel{500}}} = 0.1.$

92

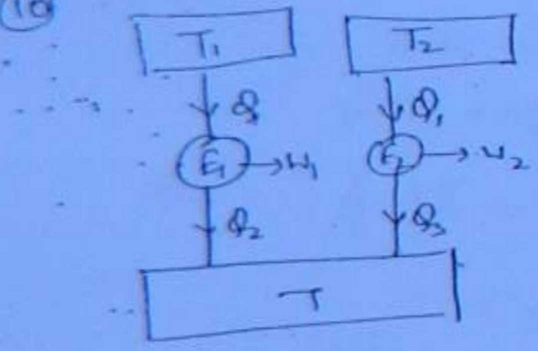
$\eta_2 = \frac{O}{I} = \frac{W}{Q_{S_2}}$

But $\eta_1 = \eta_2 \Rightarrow \frac{\cancel{W}}{Q_{S_1}} = \frac{\cancel{W}}{Q_{S_2}}$

$\therefore \underline{Q_{S_1} = Q_{S_2}}$ $\therefore Q_{R_1} = Q_{R_2}.$

∵ Supply ~~heat~~ is same, work is same and efficiency is same, therefore rejection will also be same.

⑩



$\left(\frac{T_2}{T_1 - T_2}\right)$

$\left(\frac{1}{2-1}\right) = \underline{1}.$

$\eta_1 \left(1 - \frac{T}{T_1}\right), \eta_2 \left(1 - \frac{T}{T_2}\right).$

$\eta_1 = 1 - \frac{T}{T_1} = \frac{W_1}{Q}$

$\eta_2 = 1 - \frac{T}{T_2} = \frac{W_2}{Q}$

$W_1 = \underline{Q_1 - Q_2}$

$W_2 = \underline{Q_1 - Q_3}$

$\therefore W_1 = Q\left(1 - T/T_1\right)$

$W_2 = Q\left(1 - T/T_2\right)$

∵ T_2 is less ∴ W_2 will be less.

or, $\underline{W_1 > W_2}$

⑫ $(\eta_{max})_E = \left(1 - \frac{T_2}{T_1}\right) = \left(\frac{T_1 - T_2}{T_1}\right)$

$(\eta_{max})_R = \left(\frac{T_2}{T_1 - T_2}\right) =$

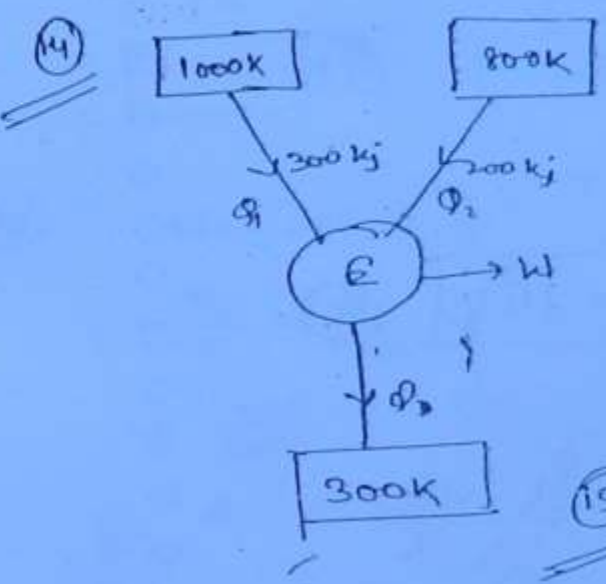
$\frac{T_1}{T_1 - T_2} - 1$

$\frac{\cancel{T_1} - \cancel{T_1} + T_2}{T_1 - T_2} = \left(\frac{T_2}{T_1 - T_2}\right)$

$$\boxed{(\eta_{max})_R = \frac{1}{(\eta_{max})_E} - 1}$$

(93)

⑭



$\frac{300}{1000} + \frac{200}{800} = \frac{Q}{300}$

$\Rightarrow 0.3 + 0.25 = \frac{Q}{300}$

$\therefore Q = 165$ kj ⓑ

⑮ $Q_1 + Q_2 = W + Q$

$\Rightarrow W = Q_1 + Q_2 - Q$

$= 500 - 165 = 335$ kj ⓓ

⑬ $T_h = 627°C = 900$ K.

$T_L = 27°C + 273 = 300$K.

$\eta_{max} = \eta_{rev} = 1 - \frac{300}{900} = 0.6667 = 66.67\%$.

$\eta_{inventor} = ?$

$\therefore \eta_{inventor} = O/I$

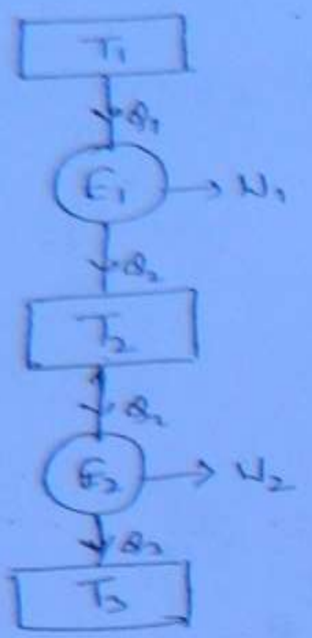
Output = 50 kW = 50 kJ/sec.

Input = $\frac{75000 \times 3}{3600}$ kj/sec = 62.5 kj/sec

$\therefore \eta_{inventor} = \frac{50}{62.5} = 0.8 = 80\%$

$\because$ Reversible efficiency is max^m efficiency and $\eta_{inventor}$ is exceeding that, hence it is not possible.

⑯



$\eta = \frac{W_1}{Q_1}$

$\eta_{E_1} = 1 - \frac{T_2}{T_1}$

$\eta_{E_2} = 1 - \frac{T_3}{T_2}$

$\frac{T_1 - T_2}{T_1} = \frac{T_2 - T_3}{T_2}$ (94)

$\Rightarrow \frac{\cancel{T_2T_1} - T_2^2 - \cancel{T_1T_2} + T_1T_3}{T_1T_2} = 0$

$\Rightarrow T_1T_3 = T_2^2$ $\therefore \boxed{T_2 = \sqrt{T_1T_3}}$ ⓐ

$W_1 = Q_1 - Q_2$

$W_2 = Q_2 - Q_3$

$W_1 = W_2 \Rightarrow Q_1 - Q_2 = Q_2 - Q_3$

$\Rightarrow$ ~~$Q_1 = Q_2 - Q_2 + Q_3$~~

$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$

$\frac{Q_2}{T_2} = \frac{Q_3}{T_3}$

$\frac{Q_1}{T_1} = \frac{Q_3}{T_3}$

$\eta_1 = \frac{W_1}{Q_1} \Rightarrow 1 - \frac{T_2}{T_1} = \frac{W}{Q_1}$

$\eta_2 = \frac{W_2}{Q_2} \Rightarrow 1 - \frac{T_3}{T_2} = \frac{W}{Q_2}$

$Q_1\left(1 - \frac{T_2}{T_1}\right) = Q_2\left(1 - \frac{T_3}{T_2}\right)$

⑰ $W_1 = W_2$ (given)

$\Rightarrow Q_1 - Q_2 = Q_2 - Q_3 \Rightarrow Q_1 + Q_3 = 2Q_2$ — ①

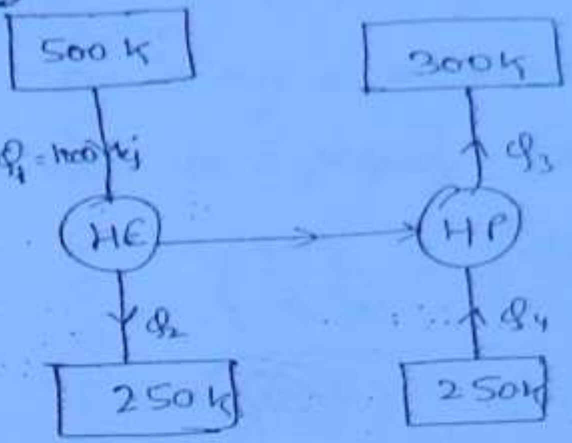
$\frac{Q_1}{T_1} = \frac{Q_2}{T_2} = \frac{Q_3}{T_3} = K \Rightarrow Q_1 = KT_1,\ Q_2 = KT_2$

$Q_3 = KT_3$

$\cancel{K}T_1 + \cancel{K}T_3 = 2\cancel{K}T_2$

$\Rightarrow T_2 = (T_1 + T_3)/2$ ⓑ

⑱ ⑲



$$(COP)_{HP} = \frac{Q_3}{W} = \frac{T_h}{T_h - T_l}$$

$$= \frac{300}{300-250} = 6.$$

①

(95)

⑱ $W = (1000 - Q_2)$.

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$

$$\Rightarrow \frac{1000}{500} = \frac{Q_2}{250}$$

$$\Rightarrow Q_2 = 500 \text{ kJ}$$

$\therefore W = (1000 - 500) = 500$ kJ

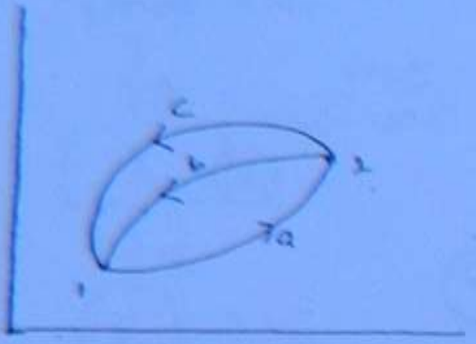
$$Q_3 = W \times 6 = 500 \times 6 = \underline{\underline{3000 \text{ kJ}}}$$

①

← : ENTROPY : →

Case - I :-

REVERSIBLE CYCLE :-



1 – a – 2 – b – 1 → reversible cycle

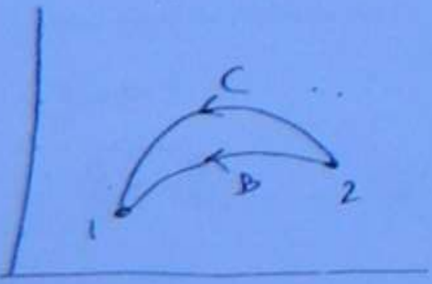
1 – a – 2 – c – 1 → reversible cycle

$$\oint_{rev} \frac{dQ}{T} = 0 \Rightarrow \cancel{\left(\frac{dQ}{T}\right)_{1a2}} + \left(\frac{dQ}{T}\right)_{2b1} = 0$$

$$\cancel{\left(\frac{dQ}{T}\right)_{1a2}} + \left(\frac{dQ}{T}\right)_{2c1} = 0$$

$$\left(\frac{dQ}{T}\right)_{2b1} - \left(\frac{dQ}{T}\right)_{2c1} = 0$$

or,
$$\boxed{\left(\frac{dQ}{T}\right)_{\substack{2b1 \\ (rev.)}} = \left(\frac{dQ}{T}\right)_{2c1\ (rev.)}}$$



$\left(\frac{dQ}{T}\right)_{rev}$ is same for both paths 'b' and 'c' and it depends only on end points (2 and 1), therefore $\left(\frac{dQ}{T}\right)_{rev}$ must be a property and this property is known as entropy.

$$\left(\frac{dQ}{T}\right)_{2b1} = \left(\frac{dQ}{T}\right)_{2c1} = ds.$$

$$\boxed{\left(\frac{dQ}{T}\right)_{\underbrace{2b1}_{rev^{***}}} = ds}$$

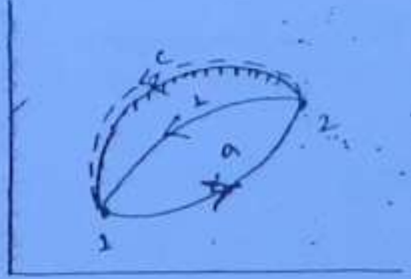
This is the second law of thermodynamics ~~for~~ equation for a process.

(97)

$$\boxed{ds \neq \left(\frac{dQ}{T}\right)^{***}_{REV}}$$

CASE-II :

IRREVERSIBLE CYCLE :-



1-a-2-b-1 (reversible cycle)
1-a-2-c-1 (irreversible ")

$$\oint_{rev} \frac{dQ}{T} = 0$$

$$\left(\frac{dQ}{T}\right)_{1a2} + \left(\frac{dQ}{T}\right)_{2b1} = 0$$

$$\left(\frac{dQ}{T}\right)_{1a2} = -\left(\frac{dQ}{T}\right)_{2b1} \quad ①$$

$$\oint_{\text{irrev.}} \frac{dQ}{T} < 0 = \left(\frac{dQ}{T}\right)_{1a3} + \left(\frac{dQ}{T}\right)_{2c1} < 0 \quad \text{— ②}$$

Substituting the value from eqⁿ ① to eqⁿ ②.

$$-\left(\frac{dQ}{T}\right)_{2b1} + \left(\frac{dQ}{T}\right)_{2c1} < 0$$

$$\underbrace{\left(\frac{dQ}{T}\right)_{2c1}}_{\text{irrev.}} \nless \underbrace{\left(\frac{dQ}{T}\right)_{2b1}}_{\text{rev.}}$$

(98)

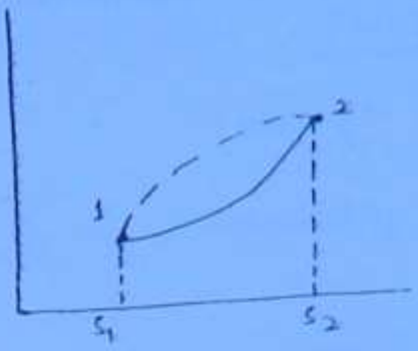
$$\left(\frac{dQ}{T}\right)_{\text{irrev.}} < ds$$

or,
$$\boxed{ds > \left(\frac{dQ}{T}\right)_{\text{irrev.}}}$$

$$ds = \left(\frac{dQ}{T}\right)_{\text{rev.}}$$

$$ds > \left(\frac{dQ}{T}\right)_{\text{irrev.}}$$

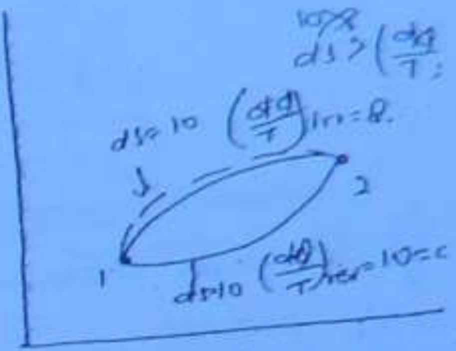
$$\therefore \boxed{ds \geq \left(\frac{dQ}{T}\right)}$$



$(dS)_{irrev} = S_2 - S_1$

$$\boxed{(dS)_{rev} = (dS)_{irrev}}$$

$(dS)_{rev} = S_2 - S_1$



As entropy is a property, it depends only on ~~po~~ end points and hence as long as the end points are same, the entropy change remains same for reversible as well as irreversible process.

In order to find the entropy change for irreversible process, it must be replaced by a reversible process between same end points.

-1 ENTROPY CHANGE OF SYSTEM FOR REVERSIBLE PROCESS:-

Case-1:- ~~When heat is app~~ When heat is supplied to the system:-

$$dS = \left(\frac{\overset{\to +ve}{dQ}}{\underset{\to +ve}{T}}\right)$$

$$dS = +ve$$

$$\underline{dS > 0.}$$

The entropy of system increases when heat is supplied in a reversible process.

Case-II:-

When heat is rejected from the system:-

$$ds = \frac{dQ^{\;-ve}}{T_{\;+ve}}$$

$$ds = -ve.$$

$$\boxed{ds < 0}$$

100

Entropy of a system decreases when heat is rejected in a reversible process.

ase-III:-

Reversible Adiabatic Process:-

$$ds = \frac{\delta Q}{T}$$

1) reversible

2) Adiabatic $\longrightarrow \delta Q = 0$

$$ds = \frac{0}{T}$$

$$ds = 0$$

$$S_2 - S_1 = 0 \quad \rightarrow \text{Isot Entropy}$$

$$S_2 = S_1 = \text{constant}.$$

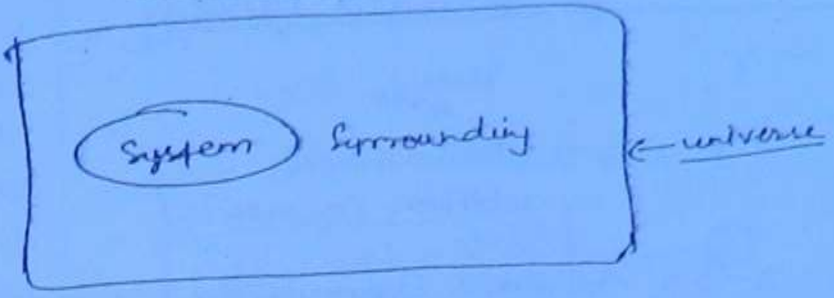
→ Isentropic

reversible adiabatic process is always isentropic process.

:-

System entropy can increase, ~~stott~~ system entropy can decrease or it can remain constant in a reversible process depending upon heat transfer.

-: ENTROPY CHANGE OF UNIVERSE :-



$$ds \geq \frac{dQ}{T}$$

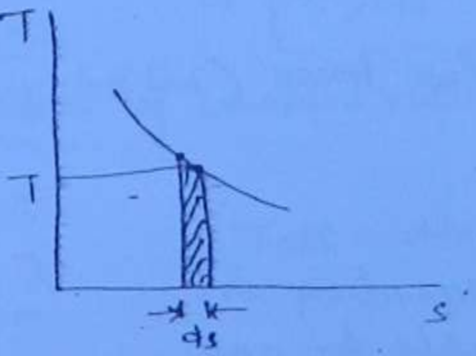
$$ds_{(universe)} \geq 0 \qquad (\because \text{universe is isolated system} \therefore dQ = 0)$$

$$(dS)_{system} + (dS)_{surrounding} \geq 0.$$

Note

Entropy of system can increase, can decrease or it can remain constant. Similarly, entropy of surroundings can increase, can decrease or can remain constant but the overall change in entropy of universe can never decrease. and this is known as the principle of increase of entropy.

← T-S DIAGRAM :→



$$\frac{dQ}{T} = dS$$

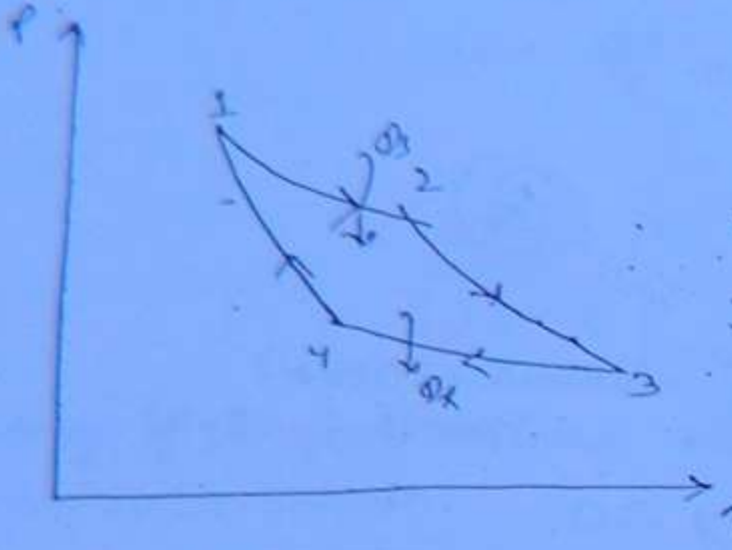
$$dQ = TdS. \quad — ②$$

$$\boxed{dQ = \text{Area}}$$

Area = Tds — ①

Area under the curve when projected on entropy axis gives reversible heat transfer.

REPRESENTATION OF CARNOT CYCLE ON T-S DIAGRAM.

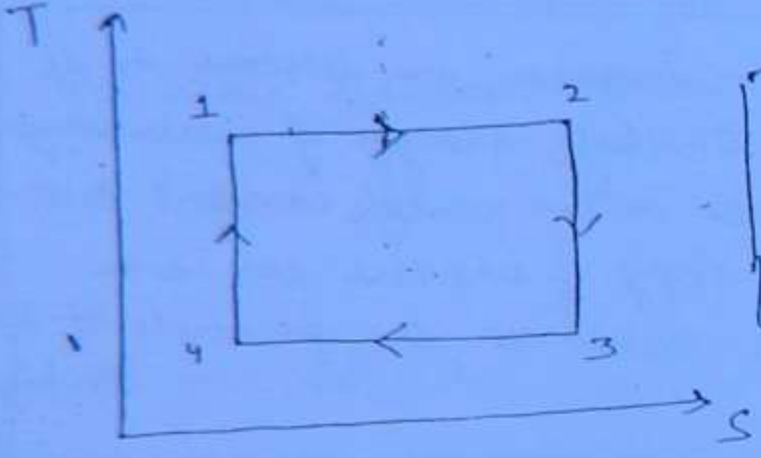


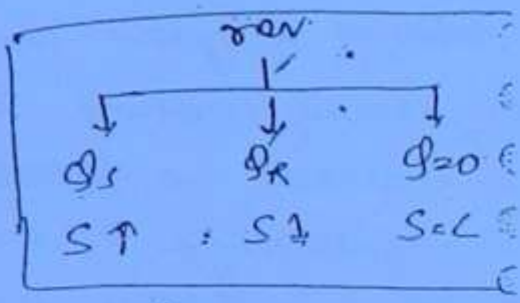
1-2 → Isothermal heat addition (expansion) → rev

2-3 → Adiabatic Expansion (rev.)

3-4 → Isothermal heat rejection (rev) (compression)

4-1 → Adiabatic comp. (rev)





COMBINED FIRST AND SECOND LAW OF THERMODYNAMICS

$dQ = dU + PdV$ → rev. process (1st law)

$\frac{dQ}{T} = dS \Rightarrow dQ = TdS$ → Rev. process (2nd law)

Combining both, we have.

$$\boxed{TdS = dU + PdV} \quad -①$$

→ This equation is applicable for any process (reversible or irreversible) because it connects various properties.

$$h = u + Pv$$

$$dh = du + Pdv + vdp$$

$$dh = dQ + vdP \qquad (\because dQ = du + Pdv)$$

$$dh = Tds + vdp \qquad (\because dQ = Tds)$$

$$\therefore \boxed{Tds = dh - vdp} \quad \text{—} ②$$

This equation is applicable for reversible as well as irreversible process. because it connects various properties

Equation ① and Eq: ② are combined first and second law equations.

⇒* REPRESENTATION OF CONSTANT-VOLUME AND CONSTANT PRESSURE LINES ON T-S DIAGRAM:- (for ideal gas):-

$$Tds = du + Pdv$$

$$v = c \Rightarrow dv = 0.$$

$$du = c_v dT$$

$$Tds = c_v dT + P(0)$$

$$\Rightarrow Tds = c_v dT$$

$$\Rightarrow \boxed{dT/ds = T/c_v}$$

Slope of constand volume lines of T-S diagram = T/c_v.

$$Tds = dh - vdp$$

$$P = c.$$

$$dP = 0$$

$$dh = c_p dT$$

$$Tds = c_p dT - v(0)$$

$$\Rightarrow Tds = c_p dT$$

$$\Rightarrow \boxed{\frac{dT}{ds} = T/c_p}$$

Hence,

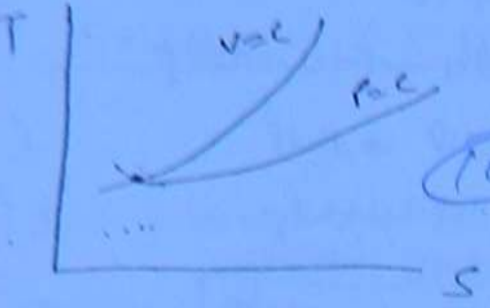
Slope of constant pressure lines on T-S diagram = T/c_p.

(P.T.O)

$P = C \rightarrow T/C_p$

$V = C \rightarrow T/C_v$

$\therefore C_p > C_v$.



As 'C_p' is greater than 'C_v' ($C_p > C_v$), the slope of constant volume lines is greater than the slope of constant pressure lines on T-S diagram. ~~Entropy change~~

→ ENTROPY CHANGE FOR IDEAL GAS ←

$$T\,dS = dU + P\,dV$$

$$ds = \frac{dU}{T} + \frac{P}{T}dV$$

ideal gas

$$dU = C_v dT; \quad PV = RT$$

$$\Rightarrow \frac{P}{T} = \frac{R}{V}$$

$$\int_{S_1}^{S_2} ds = \int_{T_1}^{T_2} \frac{C_v dT}{T} + \int_{V_1}^{V_2} \frac{R}{V} dV$$

$$\boxed{(S_2 - S_1) = C_v \ln\frac{T_2}{T_1} + R\ln\frac{V_2}{V_1}} \quad \text{—①}$$

$$T\,dS = dh - v\,dP$$

$$ds = \frac{dh}{T} - \frac{V}{T}dP$$

for ideal gas,

$$dh = C_p dT; \quad PV = RT$$

$$\Rightarrow \frac{V}{T} = R/P$$

$$\therefore ds = \frac{C_p dT}{T} - \frac{R}{P}dP$$

$$\int_{S_1}^{S_2} ds = \int_{T_1}^{T_2} \frac{C_p dT}{T} - \int_{P_1}^{P_2} \frac{R}{P} dP$$

$$\Rightarrow \boxed{(S_2 - S_1) = C_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}} \quad \text{—②}$$

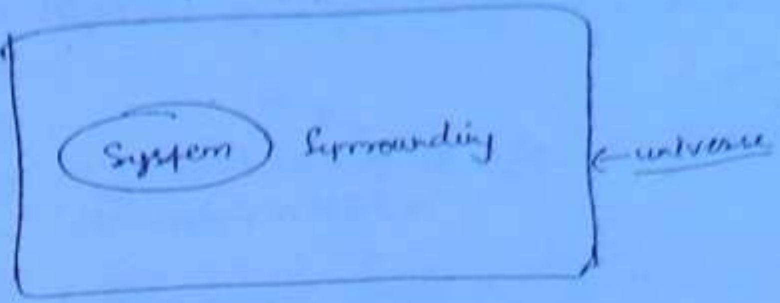
∴ Show that for an ideal gas entropy change ~~$(S_2 - S_1)$~~ =

$$(S_2 - S_1) = C_p \ln\frac{V_2}{V_1} + C_v \ln\frac{P_2}{P_1}.$$

~~$T\,dS = dh - v\,dP$~~

~~$dh = du + v\,dP + P\,dv$~~

ENTROPY CHANGE OF UNIVERSE



$$ds \geq \frac{dQ}{T}$$

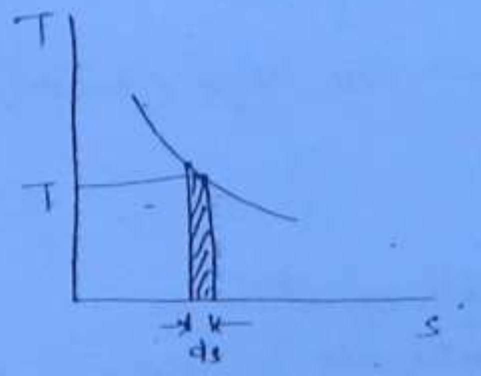
$$ds_{(universe)} \geq 0 \quad (\because \text{universe is isolated system}. \therefore dQ = 0)$$

$$(dS)_{system} + (dS)_{surrounding} \geq 0.$$

Note

Entropy of system can increase, can decrease or it can remain constant. Similarly, entropy of surroundings can increase, can decrease or can remain constant but the overall change in entropy of universe can never decrease. and this is known as the principle of increase of entropy.

← T-S DIAGRAM :→



$$\frac{dQ}{T} = dS$$

$$dQ = TdS. \quad — ②$$

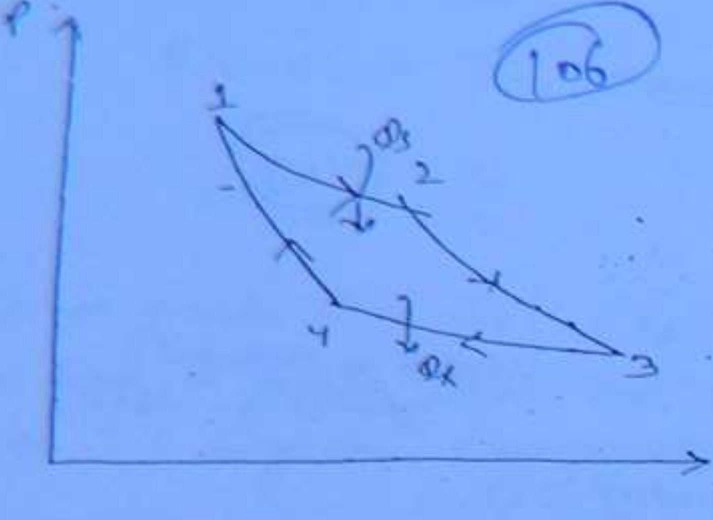
$$\boxed{dQ = Area}$$

$$Area = Tds \quad — ①$$

Area under the curve when projected on entropy axis gives reversible heat transfer.

REPRESENTATION OF CARNOT CYCLE ON T-S DIAGRAM.

106

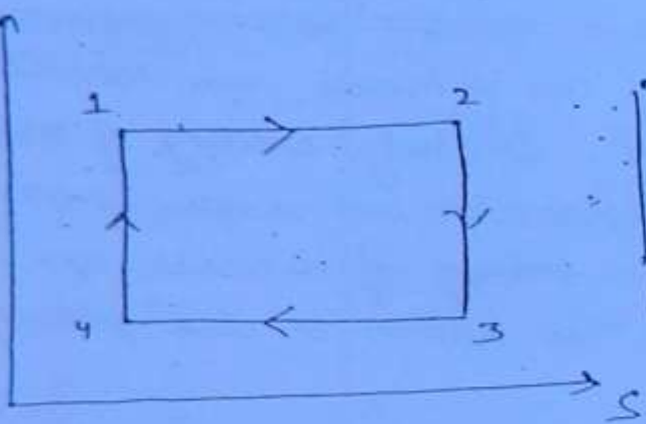


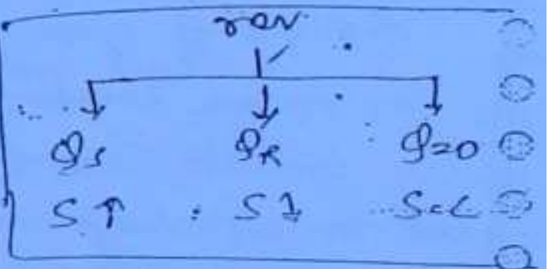
1-2 → Isothermal heat addition (expansion) rev

2-3 → Adiabatic Expansion (Rev.)

3-4 → Isothermal heat rejection (rev) (compression)

4-1 → Adiabatic comp. (rev)





COMBINED FIRST AND SECOND LAW OF THERMODYNAMICS

$dQ = dU + PdV$ → rev. process (1st law)

$\frac{dQ}{T} = dS \Rightarrow dQ = TdS$ → Rev. process (2nd law)

Combining both, we have.

$$\boxed{TdS = dU + PdV} \quad -①$$

→ This equation is applicable for any process (reversible or irreversible) because it connects various properties.

$$(S_2 - S_1) = C_p \ln \frac{V_2}{V_1} + C_v \ln \frac{P_2}{P_1}$$

Sol^n: ~~$(S_2 - S_1) = C_v \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1}$~~

Also, ~~$(S_2 - S_1) = \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$~~

~~$\Rightarrow \frac{P_1 V_1}{P_2 V_2} = \frac{T_1}{T_2}$~~

$$(S_2 - S_1) = C_v \ln \frac{T_2}{T_1} + (C_p - C_v) \ln \frac{V_2}{V_1} \qquad (\because C_p - C_v = R)$$

$$= C_v \ln \frac{T_2}{T_1} + C_p \ln \frac{V_2}{V_1} - C_v \ln \frac{V_2}{V_1}$$

$$(S_2 - S_1) = C_v \left[\ln \frac{T_2}{T_1} - \ln \frac{V_2}{V_1} \right] + C_p \ln \frac{V_2}{V_1}$$

$$(S_2 - S_1) = C_v \ln \left[\frac{T_2 / T_1}{V_2 / V_1} \right] + C_p \ln V_2 / V_1$$

$$(S_2 - S_1) = C_v \ln \left(\frac{T_2}{T_1} \cdot \frac{V_1}{V_2} \right) + C_p \ln \frac{V_2}{V_1}$$

$$PV = mRT.$$

$$\frac{PV}{T} = mR \Rightarrow \frac{PV}{T} = C$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\Rightarrow \frac{T_2}{T_1} \cdot \frac{V_1}{V_2} = \frac{P_2}{P_1}$$

$$\therefore (S_2 - S_1) = C_v \ln \frac{P_2}{P_1} + C_p \ln \frac{V_2}{V_1}$$

$$\boxed{(S_2 - S_1) = C_p \ln \frac{V_2}{V_1} + C_v \ln \frac{P_2}{P_1}}$$

$$S_2 - S_1 = C_p \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{V_2}{V_1}\right)$$

$$S_2 - S_1 = C_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right).$$

$$S_2 - S_1 = \cancel{C_p \ln\left(\frac{P_2}{P_1}\right)}\ C_p \ln\left(\frac{V_2}{V_1}\right) + C_v \ln\left(\frac{P_2}{P_1}\right).$$

Work Book

Chapter - 5

① d
② c
③ b
④ a
⑤ d
⑥ c
⑦ a
⑧ b
⑨ ~~d~~ c
⑩ ~~c~~ d
⑪ ~~a~~ d.
⑫ ~~a~~ c
⑬ a
⑭ ~~d~~ a
⑮ a
⑯ ~~b~~ d
⑰ a
⑱ b
⑲ b
⑳ d
㉑ b
㉒ d
㉓ d
㉔ b
㉕ c
㉖ a
㉗ b
㉘ a
㉙ b
㉚ b
㉛ c
㉜ a
㉝ b
㉞ d

03/08/2011.

* $dS = \left(\frac{dQ}{T}\right)_{irrev} + (\delta S)_{gen}$; $dS = \left(\frac{dQ}{T}\right)_{rev} + 0$

$\downarrow$ external $\qquad \downarrow$ internal (irreversibilities)

Entropy change occurs due to <u>external interactions</u> and <u>internal irreversibilities</u>. In a reversible process as there are no irreversibilities the entropy change occurs only due to <u>external interactions</u>.

* In an adiabatic process as there is no interaction of system with surroundings in the form of heat transfer, therefore in an adiabatic process, there is no entropy change for surroundings.

$h = u + Pv$

$dh = du + Pdv + vdp$

$dh = dQ + vdp \quad (\because dQ = du + Pdv)$

$dh = Tds + vdp \quad (\because dQ = Tds)$

$\therefore \boxed{Tds = dh - vdp} \quad — ②$

This equation is applicable for reversible as well as irreversible process. because it connects various properties

Equation ① and Eq: ② are combined first and second law equations.

⇒* REPRESENTATION OF CONSTANT-VOLUME AND CONSTANT PRESSURE LINES ON T-S DIAGRAM:- (for ideal gas):-

$Tds = du + Pdv$

$V = C \Rightarrow dV = 0.$

$du = C_v dT$

$Tds = C_v dT + P(0)$

$\Rightarrow Tds = C_v dT$

$\Rightarrow \boxed{dT/ds = T/C_v}$

Slope of constant volume lines of T-S diagram = T/C_v.

$Tds = dh - vdp$

$P = C.$

$dP = 0.$

$dh = C_p dT$

$Tds = C_p dT - v(0)$

$\Rightarrow Tds = C_p dT$

$\Rightarrow \boxed{\frac{dT}{ds} = T/C_p}$

Hence,
Slope of constant pressure lines on T-S diagram = T/C_p

(P.T.O)

$P = C \rightarrow T/C_p$

$V = C \rightarrow T/C_v$

$\therefore C_p > C_v$

As 'C_p' is greater than 'C_v' ($C_p > C_v$), the slope of constant volume lines is greater than the slope of constant pressure lines on T-S diagram. ~~Entropy change~~

→: ENTROPY CHANGE FOR IDEAL GAS :←

$$T\,dS = dU + P\,dV$$

$$\therefore ds = \frac{dU}{T} + \frac{P}{T}dV$$

ideal gas,

$$dU = C_v dT;\quad PV = RT$$

$$\Rightarrow \frac{P}{T} = \frac{R}{V}$$

$$\int_{S_1}^{S_2} ds = \int_{T_1}^{T_2} \frac{C_v dT}{T} + \int_{V_1}^{V_2} \frac{R}{V} dV$$

$$\boxed{(S_2 - S_1) = C_v \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1}} \quad \text{— ①}$$

$$T\,dS = dh - v\,dP$$

$$ds = \frac{dh}{T} - \frac{V}{T}dP$$

for ideal gas,

$$dh = C_p dT;\quad PV = RT$$

$$\Rightarrow \frac{V}{T} = R/P$$

$$\therefore ds = \frac{C_p dT}{T} - \frac{R}{P}dP$$

$$\int_{S_1}^{S_2} ds = \int_{T_1}^{T_2} \frac{C_p dT}{T} - \int_{P_1}^{P_2} \frac{R}{P} dP$$

$$\Rightarrow \boxed{(S_2 - S_1) = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}} \quad \text{— ②}$$

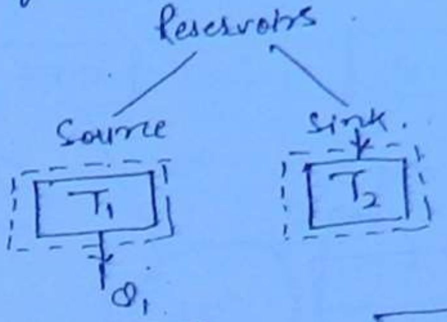
:- Show that for an ideal gas entropy change ~~$(S_2 - S_1) =$~~

$$(S_2 - S_1) = C_p \ln \frac{V_2}{V_1} + C_v \ln \frac{P_2}{P_1}.$$

~~$T\,dS = dh - v\,dP$~~

~~$dh = du + v\,dP + P\,dv$~~

* Entropy change for Reservoirs :

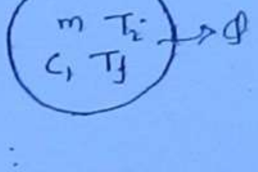


$$\boxed{(\Delta S)_{source} = -\frac{Q_1}{T_1}}$$

$$\boxed{(\Delta S)_{sink} = \frac{Q_2}{T_2}}$$

All temp must be in kelvin.

* Entropy change for a finite body :-



$$ds = \frac{dQ}{T}$$

$$\& \; dQ = mc\,dT$$

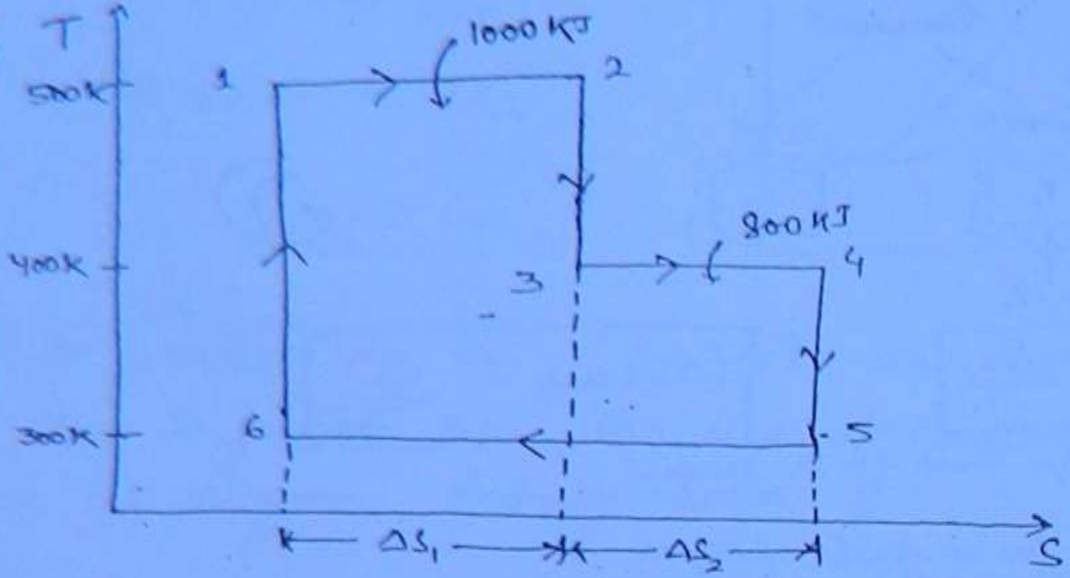
$$\int_{S_i}^{S_f} ds = \int_{T_i}^{T_f} mc \frac{dT}{T}$$

$$\Rightarrow \boxed{S_f - S_i = mc \ln \frac{T_f}{T_i}}$$

:- Problem :-

Q1. A closed system executes a reversible cycle 1-2-3-4-5-6-1 consisting of 6 processes. During process 1-2, 1000 kj of heat is added at const. tempr of 500 K. 2-3 is adiabatic expansion in which the temp decreases from 500 to 400 K. 3-4 is a constant tempr heat ~~rejection~~ addition in which 800 kj of heat is added at 400 k. 4-5 is adiabatic expansion in which the tempr decreases from 400 K to 300 K. In 5-6, heat is rejected at a constant tempr of 360 K. 6-1 is adiabatic compression. Represent the cycle on T-S diagram and calculate the net work transfer and efficiency of cycle.

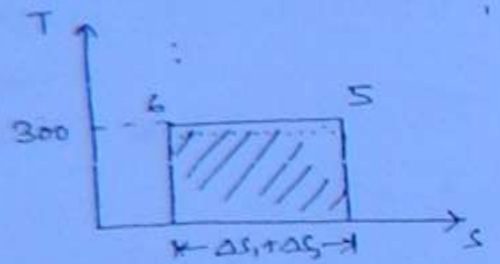
Soln:

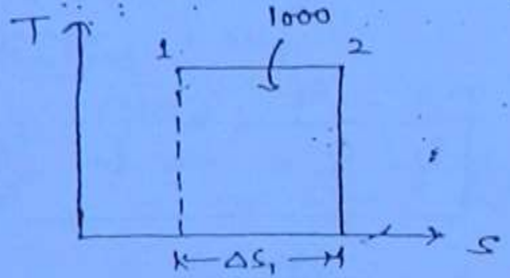


$$\Sigma W = \Sigma Q$$

$$\Rightarrow W = Q_s - Q_R$$

$$Q_s = 1000 + 800 = 1800 \text{ kj}$$





$$Q_R = 300 \times (\Delta S_1 + \Delta S_2)$$

$$= 300 \times (2+2)$$

$$= 1200.$$

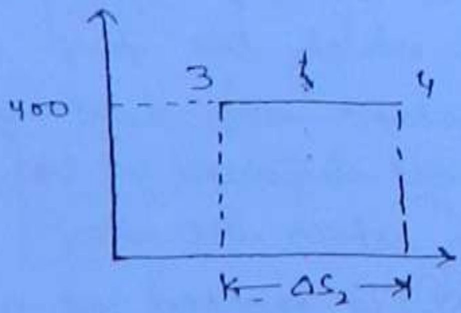
$$W = Q_s - Q_R$$

$$= 1800 - 1200$$

$$\therefore \underline{W = 600 \text{ kj}}$$

$$1000 = 500 \times \Delta S_1$$

$$\Rightarrow \underline{\Delta S_1 = 2}.$$



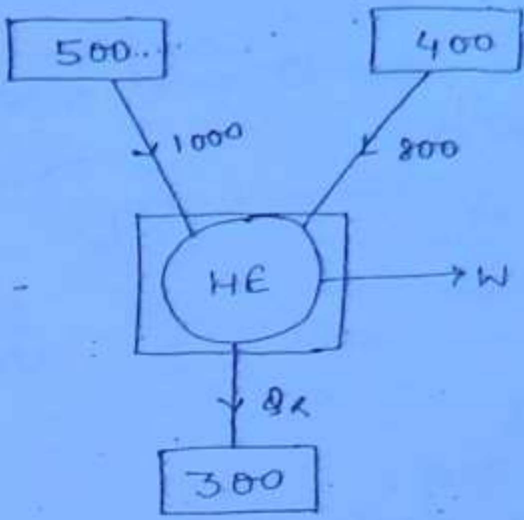
$$800 = 400 \times \Delta S_2$$

$$\therefore \underline{\Delta S_2 = 2}.$$

now efficiency,

$$\eta = W/Q_s = \frac{600}{1800} = \underline{33.3\%}.$$

Another Method :-



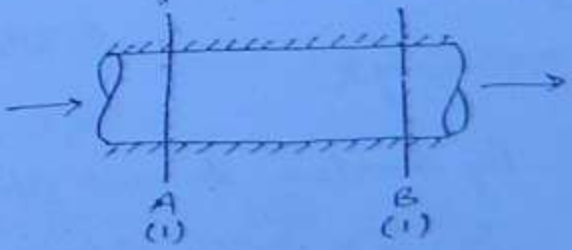
$$\oint \frac{dQ}{T} = 0 \Rightarrow \frac{1000}{500} + \frac{800}{400} - \frac{Q_R}{300} = 0.$$

$$\Rightarrow Q_R = 4 \times 300 = \underline{\underline{1200\,kJ}}.$$

Q:- Air is flowing steadily in an insulated pipe, the pressure and tempr at two stations A and B are given in table, Establish the direction of flow of air. Take $C_p = 1.005$ kJ/kg. K and $R = 0.287$ kJ/kg. K.

	A	B
Pressure	130 kPa	100 kPa
Tempr	50°C	13°C

* Pressure alone can never decide the direction. It is total energy which decides the direction



Soln: $(\Delta S)_{univ.} \geq 0$

$\Rightarrow (\Delta S)_{system} + (\Delta S)_{surr.} \geq 0$

as the system is insulated

$\Rightarrow (\Delta S)_{surr} = 0.$

$(\Delta S)_{system} \geq 0$

Let us assume, flow from A to B

Now, $(\Delta S)_{system} = S_B - S_A = C_p \ln \frac{T_B}{T_A} - R \ln \frac{P_B}{P_A}$

$= -0.0469$ kJ/kg.

$\Rightarrow (\Delta S)_{system} < 0.$

As the entropy change is less than 0, so it is not possible hence direction is from B to A.

(1/4)

Work Book

Chapter - 5

(1)(a)

adiabatic

B- Rev.

$dS = \frac{dQ}{T}$

A. Irrev.

$$dS = \left(\frac{dQ}{T}\right)_{irr} + (\delta S)_{gen}$$

$$= 0 + (\delta S)_{gen}$$

$$(\delta S)_{gen} > 0.$$

So, $\delta S > 0$.

③ $m = 10$ kg, $C_v = \frac{5R}{2}$

$P_1 = 100$
$T_1 = 300$
$P_2 = 200$
$T_2 = 500$

$C_p - C_v = R.$

$$\Rightarrow C_p = R + \frac{5}{2}R$$
$$= \frac{7}{2}R$$
$$= \frac{7}{2}(0.287)$$
$$= 1.0045 \text{ kj/kg k}$$

$$S_2 - S_1 = C_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$$
$$= 1.0045 \ln\frac{500}{300} - 0.287 \ln\frac{200}{100}$$
$$= 0.314 \text{ kj/kg k.}$$

Pay Attention to unit always.

$S_2 - S_1 = 0.314$ kj/kg-k × 10 = 3.14 kj/k.

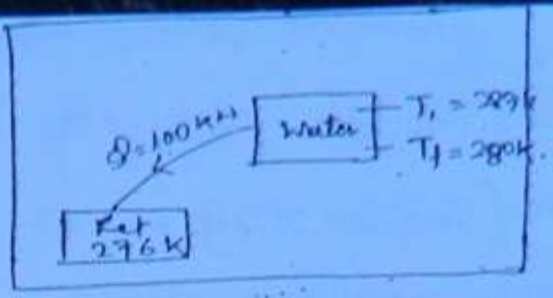
⑥

⑤ - ⓓ All are properties, hence valid for all processes.

⑦ - (c) $h = U + PV$
$= f(T) + mRT$
$h = f(T)$

ⓒ as $S_2 - S_1 = C_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$ hence, it is function of pressure also.

(7).



$Q = mc\Delta T$

$mc = \frac{Q}{\Delta T} = \frac{100}{7}$

$\Delta S = (\Delta S)_{H_2O} + (\Delta S)_{ref}$

$(\Delta S)_{ref} = \frac{Q}{T} = \frac{100}{7}$ kW/K.

$(\Delta S)_{H_2O} = mc \ln \frac{T_f}{T_1}$ 115

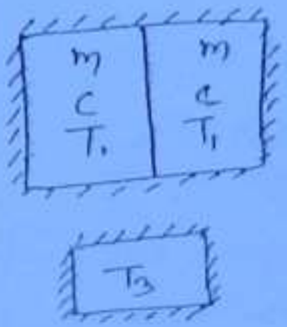
$= \frac{100}{7} \ln \frac{280}{287}$

So, $(\Delta S) = (\Delta S)_{H_2O} + (\Delta S)_{ref}$.

$= \frac{100}{7} \ln \frac{280}{287} + \frac{100}{7} \frac{kW}{K}$.

$= 9.55 \times 10^{-3}$ kW/K $= \underline{9.55 W/K}$

⑧ Show that mixing is irreversible ? (d).



$\Delta S = \Delta S_1 + \Delta S_2$ — as this is extensive

So, $\Delta S = mc \ln \frac{T_3}{T_1} + mc \ln \frac{T_3}{T_2}$

$= mc \ln \frac{T_3^2}{T_1 T_2}$

$= mc \ln \left(\frac{T_3}{\sqrt{T_1 T_2}} \right)$ —— ①

Now, heat loss = heat gain

$mc(T_1 - T_3) = mc(T_3 - T_2)$

$\Rightarrow T_1 - T_3 = T_3 - T_2$

$\Rightarrow T_3 = \frac{T_1 + T_2}{2}$

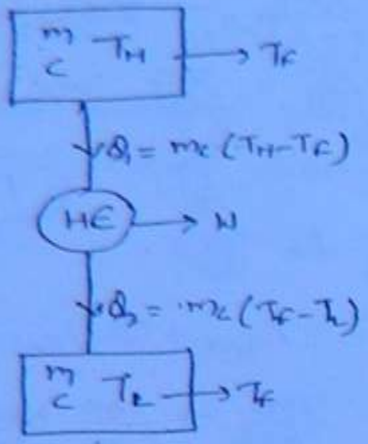
substituting it into ①, we get :-

$\Delta S = 2mc \ln \left(\frac{T_1 + T_2}{2\sqrt{T_1 T_2}} \right)$ —— ② Ans..

We know that AM > GM and hence this entropy change is (P)ve but as the system is insulated the surrounding entropy change is 0 and hence $(\Delta S)_{universe} = (\Delta S)_{system}$. Therefore in this case $(\Delta S)_{universe} > 0$ and hence mixing is an irreversible process.

$AM = \frac{T_1 + T_2}{2}$ $GM = \sqrt{T_1 T_2}$

⑨



So, $W = Q_1 - Q_2$

$$= mc\left[(T_H - T_F) - (T_F - T_L)\right]$$

$$= mc\left[T_H + T_L - 2T_F\right] \quad \text{——} ①$$

* As the system undergoes cycle the $(\Delta S)_{system} = 0$ because for a cycle initial and final points are same entropy being property depends only on points.

Now

$$(\Delta S)_{univ} \geq 0$$

$$(\Delta S)_{univ.} = 0 \quad \text{(rev. cycle)}$$

$$\underset{\text{cycle}}{\underset{\downarrow}{(\Delta S)_{system}}} + \underset{\text{surrounding}}{\underset{\downarrow}{(\Delta S)_{surrounding}}} = 0 .$$

$$0 + (\Delta S)_{surr.} = 0 .$$

$$(\Delta S)_{surr.} = 0 .$$

$$mc \ln \frac{T_F}{T_H} + mc \ln \frac{T_F}{T_L} = 0 .$$

$$\Rightarrow mc \ln \frac{T_F}{T_H} + mc \ln \frac{T_F}{T_L} = 0 .$$

$$\Rightarrow mc\left[\ln \frac{T_F^2}{T_H T_L}\right] = 0 .$$

$$\Rightarrow \ln \frac{T_F^2}{T_H T_L} = 0 = \ln 1 .$$

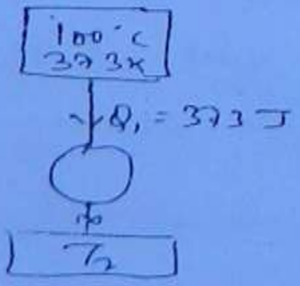
$$T_F^2 = T_H T_L$$

$$\Rightarrow T_F = \sqrt{T_H T_L}$$

and putting it into ①, we get :-

$$\boxed{W = mc\left[T_H + T_L - 2\sqrt{T_H T_L}\right]}$$

⑩ → ⓓ , ⑪ → ⓓ ⑫ − ⓒ



$$(\Delta S)_{sys} = \frac{-Q}{T}$$

$$= \frac{-373}{373}$$

$$= -1 .$$

(13) (a) Same as 8th problem.

(14) $dS = \frac{dQ}{T}$ & $dQ = mcdT$

$\Rightarrow dS = mc\frac{dT}{T}$

$\Rightarrow \int_{S_1}^{S_2} dS = m\int_{T_1}^{T_2} c\frac{dT}{T} = \int_{T_1}^{T_2} \frac{(a+bT^2)}{T}\, dT$

$\Rightarrow \int_{S_1}^{S_2} dS = \int_{T_1}^{T_2} \frac{a\,dT}{T} + \int_{T_1}^{T_2} bT\,dT$

117

$= a\ln\frac{T_2}{T_1} + b\,\frac{T_2^2 - T_1^2}{2}$

$= a\ln\frac{T_2}{T_1} + 0.5b\,(T_2^2 - T_1^2)$

Ans is (a).

(15) $Tds = dh - vdP$

$\Rightarrow TdS = dh$ as const. P.

$\Rightarrow \frac{dh}{dS} = T$

So, as T ↑ $\frac{dh}{ds}$ ↑

(a) Ans.

(16) → (d)

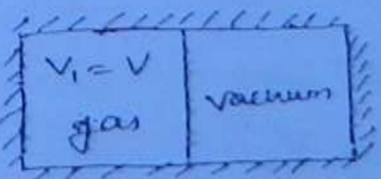
(17) $\frac{dT}{ds} = \frac{T}{C_p} = \frac{T}{\gamma R/\gamma - 1} = \frac{(\gamma - 1)T}{\gamma R}$

(d) Ans.

(18) $mc\ln\frac{T_F}{T_1} \Rightarrow 1\times1\times\ln\frac{(30+273)}{(600+273)}$

$= -1.05\ kJ/K.$

(19) (b)



$S_2 - S_1 = C_v\ln\frac{T_2}{T_1} + R\ln\frac{V_2}{V_1}$

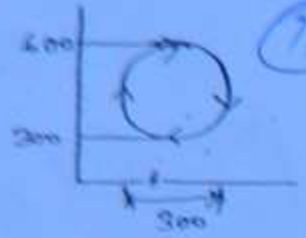
As ideal gas undergoes free expansion

So, $T_2 = T_1$

$\Delta S = C_v \ln 1 + R \ln \frac{2v}{v} = R \ln 2$ as $V_2 = 2v$

$V_1 = V$.

(20) → (a) $1 - \frac{300}{600}$



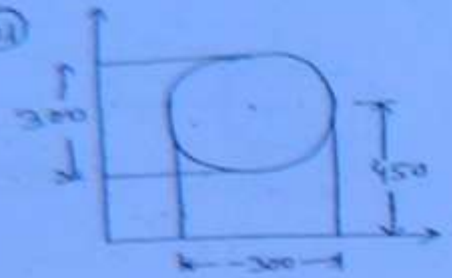
718

πr^2

$300 \times \pi \times (150)^2$ W.

$\underline{\Sigma Q = \Sigma W}$.

(21)



W = Area under curve

$= \pi/4 \, d^2 = \pi/4 \times (300)^2$

$\eta = W/Q_s$

Q_s = area under curve

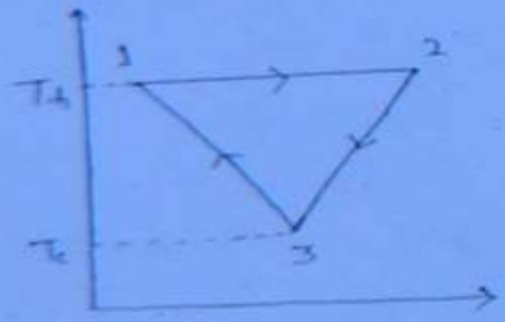
$= \frac{\pi}{4} \times (300)^2 \times 1/2 + 450 \times 300$

$$\eta = \frac{W}{Q_s} = \frac{\pi/4 \times (300)^2}{\pi/4 \times \frac{1}{2} \times (300)^2 + 450 \times 300}$$

$\eta = 41.4\%$ (Ans).

(22) - (a). (23) - (d),

(24) Increase in entropy means heat supplied. So, to find Q_s on TS diagram take the area under curve upon which 's' is increasing.



$\eta = W/Q_s$

W = Area of Δ

$= \frac{1}{2} \times (\Delta s)(T_h - T_c)$

$Q_s = T_h \times \Delta s$

(25) - (c)

(26) - (a) $S_2 - S_1 = C_v \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1}$

Rigid container V = const.

$S_2 - S_1 = C_v \ln \frac{T_2}{T_1} + R \cancel{\ln \frac{V}{V}}^{0}$

$S_2 - S_1 = C_v \ln \frac{T_2}{T_1}$. as $T_2 < T_1$.

$(\Delta S) = -ve$. Hence Ans is ⓐ.

㉗ ⓑ A − 2 During expansion there is reduction in temperature.
B − 3
C − 4
D − 5

㉘ − ⓐ As long as end points ~~are~~ are same, property are remain changed.

㉙ − Ⓑ , ㉚ − ⓑ, ㉛ − ⓒ

㉜ $S_2 - S_1 = \dot{W}/S_1$, $W = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$

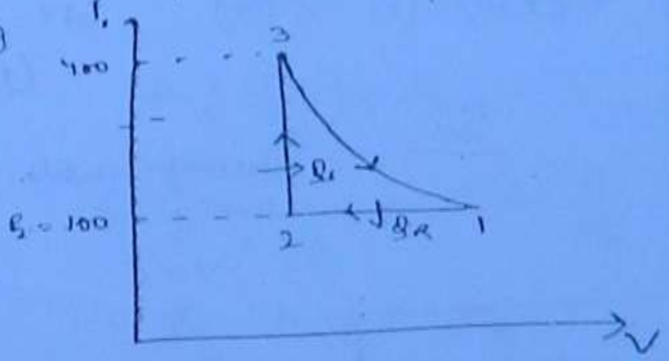
$$P_1V_1^{1.4} = P_2V_2^{1.4}$$

$$\Rightarrow \frac{400 \times 1}{100} = P_2V_2^{1.4}$$

$\ln 4 = 1.4 \ln V_2$ $= \underline{\underline{2.69\ m^3}}$

$$W = \frac{P_1V_1 - P_2V_2}{1.4 - 1} = 327.5\ kJ$$

㉝ ⓐ



$$Q_v = dU = mC_v dT$$

$$Q_P = dH = mC_p dT$$

$$\eta = 1 - \frac{Q_R}{Q_S}$$

$$Q_S = mC_v(T_3 - T_2)$$

$$Q_{12} = -mC_p(T_1 - T_2)$$

$$-Q_{12} = mC_p(T_1 - T_2)$$

$$Q_R = mC_p(T_1 - T_2)$$

$$\eta = 1 - \frac{\dot{m}c_p(T_1-T_2)}{\dot{m}c_v(T_3-T_2)}$$

$$= 1 - \frac{\gamma\,\cancel{T_2}\left(\frac{T_1}{T_2}-1\right)}{\cancel{T_2}\left(\frac{T_3}{T_2}-1\right)}$$

(120)

$$= 1 - \frac{\gamma\,(T_1/T_2 - 1)}{(T_3/T_2 - 1)}$$

$$= 1 - \frac{\gamma\left(\frac{V_1}{V_2}-1\right)}{\left(\frac{P_3}{P_2}-1\right)}$$

$$\eta = 1 - \frac{1.4\left(\frac{2.69}{1}-1\right)}{\left(\frac{400}{100}-1\right)}$$

$\therefore \eta = 0.211 = 21.1\%$

(3) - (b).

4) $P_1 = 100$ $V_1 = 0.002$

$P_2 = 125$ $V_2 = 0.003$

$$S_2 - S_1 = C_p \ln\frac{V_1}{V_2} + C_v \ln\frac{P_2}{P_1}$$

$$= 1.005 \times \ln\frac{0.003}{0.002} + 0.718 \ln\left(\frac{125}{100}\right)$$

$$= 0.567.$$

(d) Ans.

(9) - (a). Isentropic

↓

Incompressible

↓

ρ = const ⇒ V = const.

ρ = m/vol.

⇒ $V = C$

$TdS = dU + PdV$

V = const, so $dV = 0$.

$\therefore TdS = dU$

S = const (given)

$dS = 0$.

$T(0) = dU$

$\Rightarrow dU = 0 \longrightarrow dU = C(dT)$

U = const $\longrightarrow 0 = C(dT)$

[illegible] $U = C(T)$ $dT = 0$

(Ans)

So.

Incompressible

↓

$V = C$

↓

$S = C$

↓

$U = C$

↓

$T = C$

(5) → (c) (Ans).

ESE-2008

Q:- Find the irreversibility associated with the expansion of air ~~to~~ through a very small opening in a pipe from pressure and temperature of 8 bar and 600 K to a pressure of 1.2 bar. Assume it to be an ideal gas and take temperature of surroundings 298 K

Sol^n:- As the fluid is flowing through a very small opening, it undergoes throttling ($h =$ constant)

$$I = T_0 \left[(\Delta S)_{system} + (\Delta S)_{surroundings} \right].$$

$$I = T_0 (\Delta S)_{sys}.$$

(121)

$$(\Delta S)_{sys} = S_2 - S_1 = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

for throttling,

$$h_1 = h_2$$

$$\cancel{C_p} T_1 = \cancel{C_p} T_2$$

$$\Rightarrow T_1 = T_2 \quad \text{or} \quad \frac{T_2}{T_1} = 1.$$

$$\ln\left(\frac{T_2}{T_1}\right) = \ln 1 = 0.$$

$$\therefore (\Delta S)_{sys} = -R \ln \frac{P_2}{P_1} = R \ln\left(\frac{P_1}{P_2}\right)$$

$$\therefore I = T_0 (\Delta S)_{sys}$$

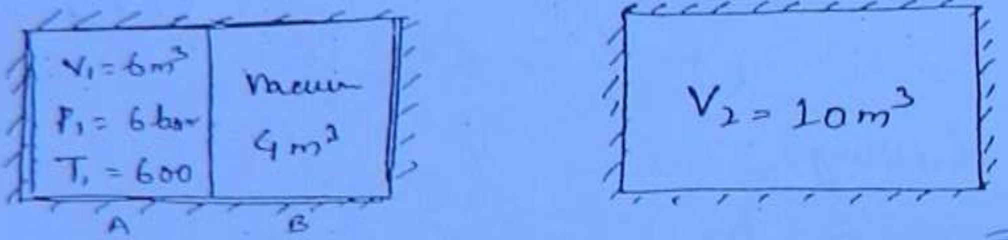
$$= T_0 \left[R \ln\left(\frac{P_1}{P_2}\right) \right].$$

$$= 298 \times 0.287 \times \ln\left(\frac{8}{1.2}\right)$$

$$\Rightarrow \boxed{I = 162.25 \text{ kj/kg}}$$

Q:- An adiabatic cylinder of 10 m³ volume is divided into two compartments 'A' and 'B', each of volume 6 m³ and 4 m³ by a thin sliding partition. Initially the compartment 'A' is filled with air at 6 bar, 600 K, while there is vacuum in compartment B. Suddenly, the partition is removed and the fluid in compartment 'A' expands freely and fills both the compartments. Calculate the irreversibility in kJ and take atmospheric pressure as 1 bar and atmospheric temperature as 300 K.

Sol:-



(122)

$$I = T_0\left[(\Delta S)_{sys} + (\Delta S)_{surr.}\right]$$

∵ The system is insulated, ∴ $(\Delta S)_{surr} = 0$.

$$\therefore I = T_0\left[(\Delta S)_{sys}\right]$$

$$(\Delta S)_{sys} = S_2 - S_1 = C_v \ln\frac{T_2}{T_1} + R\ln\frac{V_2}{V_1}$$

$T_1 = T_2$ (from free expansion)

$$\therefore \frac{T_2}{T_1} = 1 \Rightarrow \ln\left(\frac{T_2}{T_1}\right) = \ln 1 = 0.$$

$$(\Delta S)_{sys} = R\ln\frac{V_2}{V_1}$$

$$I = T_0\left(R\ln\frac{V_2}{V_1}\right) \qquad \left(K \times \frac{kJ}{kg\,K} = kJ/kg\right)$$

$$= 300 \times 0.287 \times \ln\left(\frac{10}{6}\right)$$

$$\therefore I = 43.98 \text{ kJ/kg}.$$

$PV = mRT$

$m = PV/RT = \frac{6 \times 100 \times 6}{0.287 \times 600}$

$\therefore m = 20.9$ kg,

$\therefore I = (43.98 \times 20.9) \frac{kJ}{\cancel{kg}} \times \cancel{kg}$

$\therefore \boxed{I = 919.2 \text{ kJ}}$ (Ans)

(123)

Q: A solid aluminium sphere of 0.1 m diameter and initially at 200°C is cooled to ambient temperature of 25°C. The density of aluminium is 2700 kg/m³ and specifoc heat is 0.9 kJ/kgK. find the irreversibility associated with ~~pr~~ this process in kJ.

Soln:
~~$T_1 = 200°C = 473$ K,~~
~~$T_2 = 25°C = 298$ K,~~
~~Vol^m~~ ~~Area~~ ~~$= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{0.1}{2}\right)^3 = 0.00052 \text{ m}^3$~~
~~$\rho = m/v \Rightarrow m = v \times \rho = 0.00052 \times 2700 = 1.414$ kg.~~
~~$I = T_o\left[(\Delta S)_{sys} + \cancel{(\Delta S)_{surr}}\right]$~~
~~$I = T_o\left[(\Delta S)_{sys}\right]$~~
~~$(\Delta S)_{sys} =$~~

(P.T.O)

ESE-

Q:- Steam flows steadily in an adiabatic turbine, the enthalpy at the entrance is 4142 kJ/kg and at the exit is 2585 kJ. The values of flow availability at the entrance and exit are 1787 kJ and 140 kJ/kg respectively. The ambient temperature is 300K.

(125)

find :- ① actual work

② max^m work

③ change in entropy of the system. (Neglect K.E & P.E changes).

Sol:- $W_{max} = \phi_1 - \phi_2 = 1787 - 140 = 1647$ kJ/kg.

$h_1 = 4142$ kJ/kg, $h_2 = 2585$ kJ/kg.

$\phi_1 = 1787$ kJ/kg, $\phi_2 = 140$ kJ/kg

Steady flow equation:

$$h_1 + \cancel{\frac{C_1^2}{2}} + \cancel{Z_1 g} + \cancel{Q}^{0} = h_2 + \cancel{\frac{C_2^2}{2}} + \cancel{Z_2 g} + W_{CV}$$

(adiabatic) (Neglecting K.E & P.E changes).

ⓐ $\therefore \boxed{W_{(actual)} = h_1 - h_2 = 1557 \text{ kJ/kg}}$

ⓑ $W_{max} = \phi_1 - \phi_2$ (∵ the system is open).

$= (1787 - 140) = 1647$ kJ/kg.

ⓒ $W_{max} = h_1 \; h_2 - T_0(S_1 - S_2)$ [∵ the system is open].

$\Rightarrow 1647 = 4142 - 2585 - 300(S_1 - S_2)$

$\Rightarrow (S_1 - S_2) = -0.3$ kJ/kg-K

$\therefore (\Delta S) = (S_2 - S_1) = -(S_1 - S_2) = \underline{\underline{0.3 \text{ kJ/kg K}}}$.

(Ans)

ESE -

Q.: Steam flows steadily in an adiabatic turbine, the enthalpy at the entrance is 4142 kj/kg and at the exit is 2585 kj. The values of flow availability at the entrance and exit are 1787 kj and 140 kj/kg respectively. The ambient temperature is 300K.

find :- ① actual work

② maxm work

③ change in entropy of the system. (Neglect K.E & P.E changes).

125

Sol: $W_{max} = \phi_1 - \phi_2 = 1787 - 140 = 1647 \text{ kj/kg}$

$h_1 = 4142 \text{ kj/kg}, \quad h_2 = 2585 \text{ kj/kg}$

$\phi_1 = 1787 \text{ kj/kg}, \quad \phi_2 = 140 \text{ kj/kg}$

Steady flow equation:

$$h_1 + \cancel{\frac{c_1^2}{2}} + \cancel{z_1 g} + \overset{0}{\cancel{Q}} = h_2 + \cancel{\frac{c_2^2}{2}} + \overset{0}{\cancel{z_2 g}} + W_{cv}$$

(adiabatic)

(Neglecting K.E & P.E changes).

ⓐ $\therefore \boxed{\underset{(actual)}{W} = h_1 - h_2 = 1557 \text{ kj/kg}}$

ⓑ $W_{max} = \phi_1 - \phi_2$ (∵ The system is open).

$= (1787 - 140) = 1647 \text{ kj/kg}$

ⓒ $W_{max} = h_1 - h_2 - T_0(S_1 - S_2)$ [∵ The system is open]

$\Rightarrow 1647 = 4142 - 2585 - 300(S_1 - S_2)$

$\Rightarrow (S_1 - S_2) = -0.3 \text{ kj/kg-K}$

$\therefore (\Delta S) = (S_2 - S_1) = -(S_1 - S_2) = \underline{0.3 \text{ kj/kg K}}$

(Ans)

ESE -

Q: Steam flows steadily in an adiabatic turbine, the enthalpy at the entrance is 4142 kj/kg and at the exit is 2585 kj. The values of flow availability at the entrance and exit are 1787 kj and 140 kj/kg respectively. The ambient temperature is 300K.

(125)

find :- ① actual work

② maxm work

③ change in entropy of the system. (Neglect K.E & P.E changes).

Sol: $W_{max} = \phi_1 - \phi_2 = 1787 - 140 = 1647$ kj/kg.

$h_1 = 4142$ kj/kg, $h_2 = 2585$ kj/kg.

$\phi_1 = 1787$ kj/kg, $\phi_2 = 140$ kj/kg

Steady flow equation:

$$h_1 + \cancelto{0}{\frac{C_1^2}{2}} + \cancelto{0}{Z_1 g} + \cancelto{0}{Q} = h_2 + \cancelto{0}{\frac{C_2^2}{2}} + \cancelto{0}{Z_2 g} + W_{CV}$$

(adiabatic)

(Neglecting K.E & P.E changes).

ⓐ $\therefore \boxed{\underset{(actual)}{W} = h_1 - h_2 = 1557 \text{ kj/kg}}$

ⓑ $W_{max} = \phi_1 - \phi_2$ (∵ The system is open).

$= (1787 - 140) = 1647$ kj/kg.

ⓒ $W_{max} = h_1 \; h_2 - T_0 (S_1 - S_2)$ [∵ The system is open].

$\Rightarrow 1647 = 4142 - 2585 - 300 (S_1 - S_2)$

$\Rightarrow (S_1 - S_2) = - 0.3$ kj/kg-K.

$\therefore (\Delta S) = (S_2 - S_1) = -(S_1 - S_2) = 0.3$ kj/kg K.

(Ans)

Find the maximum work per kg of air that can be obtained in a piston cylinder arrangement. Air expands from initial state of 9 bar, 400 K to a final state of 1.5 bar, 300 K. Assume atmospheric pressure to be 1 bar and atmospheric temperature to be 288 K. Also find maximum useful work.

(126)

$$W_{max} = U_1 - U_2 - T_0(S_1 - S_2)$$ [for closed system].

∵ Air can be treated as ideal gas.

$\therefore U = mC_vT \quad \because m = 1$ kg.

$\therefore U = C_vT.$

$$\therefore W_{max} = C_vT_1 - C_vT_2 - T_0(S_1 - S_2)$$

$$= C_v(T_1 - T_2) - T_0(S_1 - S_2)$$

Now.

$P_1 = 9$ bar $\quad T_1 = 400$ K

$P_2 = 1.5$ bar $\quad T_2 = 300$ K.

$T_0 = 288$ K.

Now,

$$(S_2 - S_1) = C_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$$

$$= 1.005\ln\left(\frac{300}{400}\right) - 0.287\ln\left(\frac{1.5}{9}\right)$$

$\therefore (S_2 - S_1) = 0.225$ kJ/kgK.

$\therefore (S_1 - S_2) = -0.225$ kJ/kg K.

$$\therefore W_{max} = C_v(T_1 - T_2) - T_0(S_1 - S_2)$$

$$= 0.718(400 - 300) - 288(-0.225)$$

$$\Rightarrow \boxed{W_{max} = 136.6\ kj/kg}$$ (Ans)

Maximum useful work,

$W_{max\ useful} = W_{max} - W_{atm}$.

$W_{atm} = P_0(V_2 - V_1)$

Now, $PV = mRT$

$$\Rightarrow V_1 = \frac{mRT_1}{P_1} = \frac{1\times0.287\times400}{9\times100} = 0.1275\,m^3$$

Similarly

$$V_2 = \frac{mRT_2}{P_2} = \frac{1\times0.287\times300}{1.5\times100} = 0.574\,m^3$$

$$\therefore W_{atm} = \cancel{1\times100}\ P_0(V_2 - V_1)$$

$$= 1\times100(0.574 - 0.1275)$$

$$= 44.65\ kj/kg$$

$$\therefore W_{max\ useful} = W_{max} - W_{atm}$$

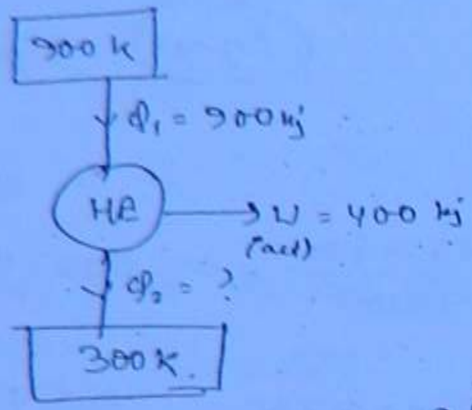
$$= (136.6 - 44.65)\ kj/kg$$

$$\boxed{W_{max\ useful} = 91.95\ kj/kg.}$$ (Ans)

Q:- A heat engine operates between two thermal reservoirs at 900 K and 300 K. The heat supplied to the engine is 900 kj. The work output is 400 kj. Find the irreversibility. Surrounding temp is 300 K. (128)

V.I

impr



$$I = T_0 \left[(\Delta S)_{sys} + (\Delta S)_{surr} \right].$$

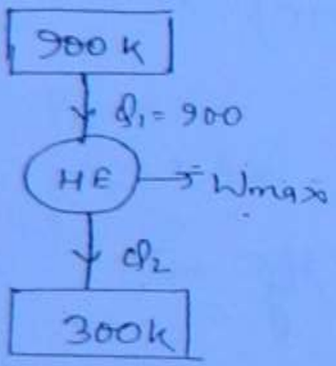
$W = Q_1 - Q_2$

$\Rightarrow Q_2 = Q_1 - W = 500$ kj

$W_{act} = 400$ kj

$I = W_{max} - W_{act}$.

Let us assume the engine to be reversible.



W_{max} is only for reversible engine.

for reversible engine,

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$

$$\Rightarrow \frac{900}{900} = \frac{Q_2}{300}$$

$\Rightarrow Q_2 = 300$ kj

$\therefore W_{max} = Q_1 - Q_2 = (900 - 300) = 600$ kj

Now, $I = W_{max} - W_{act}$

$= (600 - 400)$

$\therefore \boxed{I = 200 \text{ kj}}$ (Ans)

→ MIXTURE OF IDEAL GASES ←

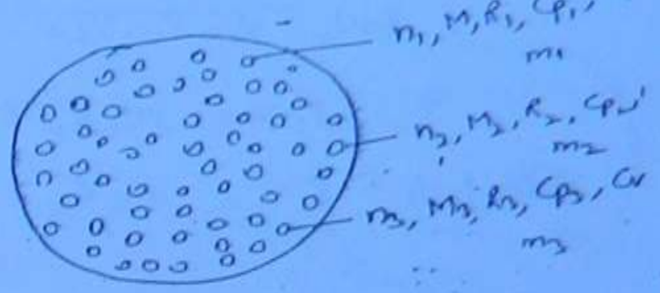
⇒ **Mole fraction (x)** :- It is defined as the ratio of no. of moles of a gas to the total number of moles.

$$x = \frac{n}{\Sigma n}$$

$$x_1 = \frac{n_1}{n_1 + n_2 + \cdots}$$

$$x_2 = \frac{n_2}{n_1 + n_2 + \cdots}$$

$$\boxed{x_1 + x_2 + \cdots = 1}$$



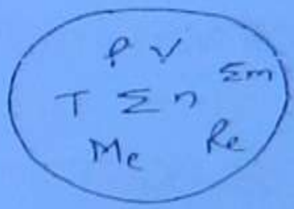
$V_1 = V_2 = V_3 = V$

$T_1 = T_2 = T_3 = T$

$P = P_1 + P_2 + P_3$

$PV = mRT$

$PV = n\bar{R}T$



$P_1 V_1 = n_1 \bar{R} T_1$

but $V_1 = V_2 = V_3 = V$ & $T_1 = T_2 = T_3 = T$

$\therefore P_1 V = n_1 \bar{R} T$ — ①

$PV = \Sigma n \bar{R} T$ — ②

Dividing ① by ② we have

$$\frac{P_1 \cancel{V}}{P \cancel{V}} = \frac{n_1 \cancel{\bar{R}T}}{\Sigma n \cancel{\bar{R}T}} \quad \therefore \quad \frac{P_1}{P} = \frac{n_1}{\Sigma n}$$

But, $\frac{n_1}{\Sigma n} = x_1$

$$\therefore \frac{P_1}{P} = x_1$$

∴ Partial pressure = Mole fraction × Total pressure

or $P_1 = x_1 P$

Similarly, $P_2 = x_2 P$

⋮

⇒ * EQUIVALENT GAS CONSTANT (R_e) :→

$$PV = mRT$$

$$P_1V_1 = m_1R_1T_1$$

But $V_1 = V_2 = V_3 = \cdots = V$ & $T_1 = T_2 = \cdots = T$.

$$\therefore P_1V = m_1R_1T$$

Similarly $P_2V = m_2R_2T$

$$\vdots$$

P V
Σn Σm
M_e R_e T

$$(P_1+P_2+\cdots)V = (m_1R_1+m_2R_2+\cdots)T$$

But, $(P_1+P_2+\cdots) = P$.

$$\therefore PV = (m_1R_1+m_2R_2+\cdots)T \quad \text{—①}$$

But

$$PV = \Sigma m R_e T \quad \text{—②}$$

from ① & ②.

$$\Sigma m R_e \cancel{T} = (m_1R_1+m_2R_2+\cdots)\cancel{T}$$

$$\therefore R_e = \frac{m_1R_1+m_2R_2+\cdots}{\Sigma m}$$

or,

$$\boxed{R_e = \frac{m_1R_1+m_2R_2+\cdots}{m_1+m_2+\cdots}}$$

Similarly,

$$\boxed{C_{pe} = \frac{m_1C_{p_1}+m_2C_{p_2}+\cdots}{m_1+m_2+\cdots}}$$

$$\boxed{C_{v_e} = \frac{m_1 C_{v_1} + m_2 C_{v_2} + \cdots}{m_1 + m_2 + \cdots}}$$

⇒ * EQUIVALENT MOLECULAR WEIGHT (M_e) :-

P V

T Σm

Σn Re

Me

$$n = \frac{m}{M}$$

or ~~$m = N$~~

$$m = nM$$

$$m_1 = n_1 M_1$$

$$m_2 = n_2 M_2$$

$$\vdots$$

$$m_1 + m_2 + \cdots = n_1 M_1 + n_2 M_2 + \cdots$$

$$\Rightarrow \Sigma m = (n_1 M_1 + n_2 M_2 + \cdots) \text{ --- } ①$$

Also,

$$\Sigma m = \Sigma n M_e \text{ --- } ②$$

from ① & ②

$$\Sigma n M_e = (n_1 M_1 + n_2 M_2 + \cdots)$$

$$\Rightarrow M_e = \frac{(n_1 M_1 + n_2 M_2 + \cdots)}{\Sigma n}$$

or, $$M_e = \frac{n_1}{\Sigma n} \cdot M_1 + \frac{n_2}{\Sigma n} \cdot M_2 + \cdots$$

$$\boxed{M_e = x_1 M_1 + x_2 M_2 + \cdots}$$

⇒ Equations to be Remembered :-

1. $PV = mRT$; $PV = n\bar{R}T$

2. $x = \frac{n}{\Sigma n}$; 3. $P_1 = x_1 P$; $P_2 = x_2 P$. and so on

4. $$R_e = \frac{m_1R_1 + m_2R_2 + \cdots}{m_1 + m_2 + \cdots}$$

5. $$C_{pe} = \frac{m_1C_{p_1} + m_2C_{p_2} + \cdots}{m_1 + m_2 + \cdots}$$

6. $$C_{ve} = \frac{m_1C_{v_1} + m_2C_{v_2} + \cdots}{m_1 + m_2 + \cdots}$$

7. $M_e = x_1M_1 + x_2M_2 + \cdots$

8. $R = \frac{\bar{R}}{M}$.

Q:- A mixture of ideal gases consists of 3 kg of nitrogen and 5 kg of Carbon dioxide at a pressure of 300 kPa and temperature of 20°C. Find ① Mole fraction of each constituent

② Equivalent molecular weight of the mixture

③ Equivalent gas constant

④ Partial pressure of each constituent

⑤ Total volume of the mixture

⑥ Density of the mixture

⑦ 'Cp' and 'Cv' of the mixture

⑧ If this mixture is heated at constant volume to 40°C, find changes in internal energy, enthalpy and entropy of the mixture.

(7) If the mixture is heated at constant pressure to 40°C, find changes in internal energy, enthalpy and entropy of the mixture.

Soln: Take 'γ' for nitrogen as 1.4 & 'γ' for CO_2 as 1.286

$N_2 - 1$; $CO_2 \rightarrow 2$.

$m_1 = 3\,kg$, $m_2 = 5\,kg$.

$M_1 = 28$; $M_2 = 44$.

$$n = \frac{m}{M} \quad \therefore \quad n_1 = \frac{m_1}{M_1} = \frac{3}{28} = 0.~~107142~~$$

$$n_2 = \frac{m_2}{M_2} = \frac{5}{44} =$$

Now,

① $x = \frac{n}{\Sigma n}$; $\therefore x_1 = \frac{n_1}{n_1+n_2} = \frac{3/28}{3/28 + 5/44} =$

$$\therefore \boxed{x_1 = 0.485}$$

$$x_2 = \frac{n_2}{n_1+n_2} = \frac{5/44}{3/28 + 5/44} = \boxed{0.515}$$ (Ans)

② $M_e = x_1 M_1 + x_2 M_2$

$= 0.485 \times 28 + 0.515 \times 44 =$

$\Rightarrow \boxed{M_e = 36.25}$

③ $R_e = \frac{\bar{R}}{M_e} = \frac{8.314}{36.25}$

$\boxed{R_e = 0.229\ kJ/kgK}$

④ $P_1 = x_1 P \Rightarrow P_1 = 0.485 \times 300$

$\Rightarrow \boxed{P_1 = 145.5\ kPa}$

$P_2 = x_2 P \Rightarrow P_2 = 0.515 \times 300 \Rightarrow \boxed{P_2 = 154.5}$

⑤ $PV = \Sigma m R_c T$

$$\Rightarrow V = \frac{8 \times 0.229 \times 293}{300}$$

$$\therefore \boxed{V = 1.789 \text{ m}^3}$$

⑥ Density, $\rho = \frac{\Sigma m}{V} = \frac{8}{1.789} = 4.47 \text{ kg/m}^3$.

$$\therefore \boxed{\rho = 4.47 \text{ kg/m}^3}$$

⑦ $$C_{pc} = \frac{m_1 C_{p_1} + m_2 C_{p_2}}{m_1 + m_2}$$

$$C_{p_1} = \frac{\gamma R}{\gamma - 1}$$

$$R_1 = \frac{\bar{R}}{M_1} = \frac{8.314}{28} = 0.2969$$

$$\Rightarrow C_{p_1} = \frac{1.4 \times 0.2969}{(1.4 - 1)}$$

$$\therefore C_{p_1} = 1.039$$

$$R_2 = \frac{8.314}{44} = 0.18895$$

$$C_{p_2} = \frac{1.286 \times .189}{1.286 - 1}$$

$$\therefore C_{p_2} = 0.849 \simeq 0.85$$

$$\therefore C_{pc} = \frac{3 \times 1.039 + 5 \times 0.849}{8}$$

$$\boxed{C_{pc} = 0.92}$$ (Ans)

Now,

$C_{pe} - C_{ve} = R_e$. $[\because C_p - C_v = R]$

$\Rightarrow C_{ve} = C_{pe} - R_e$

$= (0.92 - 0.229)$

$\therefore \boxed{C_{ve} = 0.6916 \text{ kj/kgK.}}$ (135)

⑧. $T_2 = 40°C$, $T_1 = 20°C$

(a) $dU = m C_{ve} dT$

$= 8 \times 0.6916 \times 20 = \underline{\underline{110.656 \text{ kj}}}$

(b) $dH = m C_{pe} dT$

$= 8 \times 0.9206 \times 20$

$\Rightarrow \underline{dH = 147.296 \text{ kj}}$

(c) $dS = (S_2 - S_1) = C_v \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1}$

$\because$ vol^n is contant, $\therefore V_1 = V_2$

$\therefore \ln \frac{V_2}{V_1} = \ln 1 = 0.$

$\therefore (S_2 - S_1) = C_v \ln \frac{T_2}{T_1}$

$= 0.6916 \ln \left(\frac{313}{293}\right)$

$\therefore dS = 0.0459 \text{ kj/kg K}$

$\therefore$ Total Entropy change $= (0.045 \times 8)$ kj/k

$\underline{\underline{dS =}} \boxed{0.364 \text{ kj/k}}$

(9) $dU = mC_v dT = 110.656$ kj

$dH = mC_p dT = 147.296$ kj

Now. $dS = (S_2 - S_1) = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$

$\because$ Pressures are constant.

$\therefore P_1 = P_2$.

$\therefore \ln \frac{P_2}{P_1} = \ln 1 = 0$.

$\therefore (S_2 - S_1) = C_p \ln \frac{T_2}{T_1} = 0.9206 \times \ln \left(\frac{313}{293}\right)$

$\therefore dS = (S_2 - S_1) = 0.0607$ kj/kg k

$\therefore$ Total Entropy change $= 8 \times 0.0607$

$$\boxed{\underset{(\text{Total})}{dS} = 0.486 \text{ kj/k}}$$ (Ans)

⇒ THERMODYNAMIC RELATIONS :-

Theorem :- 1 The equation $(dz = M dx + N dy)$ is exact when

$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$$

Theorem :- 2

$$f = \phi(x, y, z)$$

$$\left(\frac{\partial x}{\partial y}\right)_f \left(\frac{\partial y}{\partial z}\right)_f \left(\frac{\partial z}{\partial x}\right)_f = 1.$$

Theorem :- 3.

$$z = \phi(x, y)$$

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1.$$

⇒ * MAXWELL'S EQUATIONS :→

① $Tds = dU + PdV$

$dU = Tds - PdV$

$dz = Mdx + Ndy$

$M = T;\ x = S;\ N = -P;\ y = V.$

(137)

$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$$

$$\therefore \boxed{\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V}$$

This is first Maxwell's equation.

② $TdS = dh - VdP$

$\Rightarrow dh = TdS + VdP$

$dz = Mdx + Ndy$

$M = T;\ x = S;\ N = V;\ y = P$

$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$$

$$\therefore \boxed{\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P}$$

This is second Maxwell's equation

③ $G = H - TS$ (Gibb's function).

$dG = dH - (Tds + SdT)$

$\left[\because Tds = dH - VdP \right.$
$\left.\Rightarrow dH - Tds = VdP\right]$

or, $dG = \underbrace{dH - Tds} - SdT$

$\therefore dG = VdP - SdT$

$dz = Mdx + Ndy$

$M = V;\ x = P;\ N = -S;\ y = T.$

$(\partial M/\partial y)_x = (\partial N/\partial x)_y$

$$\therefore \boxed{\left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial S}{\partial P}\right)_T}$$

This is third Maxwell's equation

④ From Halmond's function :

~~F =~~

$$F = U - TS$$

(138)

$$dF = dU - (TdS + SdT)$$

$$\Rightarrow dF = \underbrace{dU - TdS} - SdT \quad \cdots \left[\begin{array}{l} TdS = dU + PdV \\ \Rightarrow dU - TdS = -PdV \end{array} \right)$$

$$\Rightarrow dF = -PdV - SdT$$

$$dz = Mdx + Ndy \qquad ,\ M = -P;\ x = V;\ N = -S;\ y = T.$$

$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$$

$$\therefore \boxed{\cancel{-}\left(\frac{\partial P}{\partial T}\right)_V = \cancel{-}\left(\frac{\partial S}{\partial V}\right)_T}$$

$$\Rightarrow \boxed{\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T}$$

This is 4th Maxwell's equation.

Maxwell's Equation :-

① $\boxed{\left(\frac{\partial T}{\partial V}\right)_S = \left(-\frac{\partial P}{\partial S}\right)_V}$

② $\boxed{\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P}$

③ $\boxed{\left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial S}{\partial P}\right)_T}$

④ $\boxed{\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T}$

⇒* Tds Equations :-

$S = f(T, v)$

$$ds = \left(\frac{\partial s}{\partial T}\right)_v dT + \left(\frac{\partial s}{\partial v}\right)_T dv$$ (13a)

$$Tds = T\left(\frac{\partial s}{\partial T}\right)_v dT + T\left(\frac{\partial s}{\partial v}\right)_T dv \quad \text{———} ①$$

Now At constant pressure, i.e. $P = C$

$$\frac{dT}{ds} = T/C_p$$

$$\left(\frac{\partial T}{\partial s}\right)_p = T/C_p$$

$$C_p = T\left(\frac{\partial s}{\partial T}\right)_p \quad \text{———} ⓐ$$

Similarly, $C_v = T\left(\frac{\partial s}{\partial T}\right)_v$ ——— ⓑ

Also, from Maxwell's Equation :-

$$\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial P}{\partial T}\right)_v \quad \text{———} ⓒ$$

$$\& \quad \left(\frac{\partial s}{\partial P}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_P \quad \text{—} ⓓ$$

Substituting from ⓐ & ⓒ in eq: ①, we have -

$$\boxed{Tds = C_v dT + T\left(\frac{\partial P}{\partial T}\right)_v dv} \quad \text{———} Ⓐ$$

This is known as first Tds Equation

$S = f(T,P)$

$$dS = \left(\frac{\partial s}{\partial T}\right)_P dT + \left(\frac{\partial s}{\partial P}\right)_T dP \quad \ldots$$

$$TdS = T\left(\frac{\partial s}{\partial T}\right)_P dT + T\left(\frac{\partial s}{\partial P}\right)_T dP \quad \text{—} ②$$

substituting from ③ & ④ in eq^n ②, we have :-

$$\boxed{TdS = C_p dT - T\left(\frac{\partial V}{\partial T}\right)_P dP} \quad \text{—} (B)$$

This is known as second <u>Tds</u> equation.

<u>Now</u>.

<u>Equating both T-ds equations A & B</u>, we have

$$TdS = C_v dT + T\left(\frac{\partial P}{\partial T}\right)_V dV = C_p dT - T\left(\frac{\partial V}{\partial T}\right)_P dP$$

or, $T\left(\frac{\partial V}{\partial T}\right)_P dP + T\left(\frac{\partial P}{\partial T}\right)_V dV = (C_p - C_v) dT$

or, $dT = \left(\frac{T}{(C_p - C_v)}\left(\frac{\partial V}{\partial T}\right)_P\right) dP + \left(\frac{T}{(C_p - C_v)}\left(\frac{\partial P}{\partial T}\right)_V\right) dV$ — ①

$T = \phi(P,V)$

$$dT = \left(\frac{\partial T}{\partial P}\right)_V dP + \left(\frac{\partial T}{\partial V}\right)_P dV \quad \text{—} ②$$

Comparing eq^n ① & ②, we have :-

$$\frac{T}{C_p - C_v}\left(\frac{\partial V}{\partial T}\right)_P = \left(\frac{\partial T}{\partial P}\right)_V$$

$$(C_p - C_v) = T\left(\frac{\partial V}{\partial T}\right)_P \boxed{\left(\frac{\partial P}{\partial T}\right)_V} \quad \text{—} ③$$

Again $T = \phi(P, V)$ (141)

from third theorem, we have :-

$$\left(\frac{\partial P}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial T}{\partial P}\right)_V = -1$$

$$\text{or, } -\left(\frac{\partial P}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P = \left(\frac{\partial P}{\partial T}\right)_V \quad \text{—} ④$$

Substituting eqⁿ ④ in eqⁿ ③ we have :-

$$(C_p - C_v) = T\left(\frac{\partial V}{\partial T}\right)_P \left[-\left(\frac{\partial P}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P\right]$$

$$\boxed{C_p - C_v = -T\left[\left(\frac{\partial V}{\partial T}\right)^2_P\right]\left(\frac{\partial P}{\partial V}\right)_T} \quad \text{—} Ⓒ$$

This equation has been asked 9 times.

$$C_p - C_v = -\overset{\oplus ve}{T}\left[\overset{\oplus ve}{\left(\frac{\partial V}{\partial T}\right)^2_P}\right]\overset{\ominus ve}{\left(\frac{\partial P}{\partial V}\right)_T}$$

$$\boxed{\frac{dP}{dV} = -\frac{P}{V}}$$

$C_p - C_v = +ve.$

or, $C_p - C_v > 0$

$$\boxed{C_p > C_v} \quad \text{—} Ⓓ$$

for an isothermal process $\left(\frac{\partial P}{\partial V}\right)$ is always negative. Square of a quantity is always positive. Temperature can never be negative.

⇒* CO-EFFICIENT OF VOLUME EXPANSIVITY (β):-

It shows variation of volume with respect to temperature under isobaric conditions.

$$\boxed{\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P} \quad \text{——} ⓔ$$

142

⇒* ISOTHERMAL COMPRESSIBILITY (K_T):-

This shows variation of volume with respect to pressure under isothermal conditions.

$$\boxed{K_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T} \quad \text{——} ⓕ$$

**

$$K_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T \Rightarrow \left(\frac{\partial P}{\partial V}\right)_T = -\frac{1}{K_T V}$$

Again $\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P \Rightarrow \left(\frac{\partial V}{\partial T}\right)_P = \beta V$

Now

$$C_P - C_V = -T\left[\left(\frac{\partial V}{\partial T}\right)_P\right]^2 \left(\frac{\partial P}{\partial V}\right)_T$$

$$C_P - C_V = -T\left[\beta^2 V^{\cancel{2}}\right] \times \frac{-1}{K_T \cancel{V}}$$

$$\boxed{C_P - C_V = \frac{TV\beta^2}{K_T}} \quad \text{——} ⓖ$$

****

This has been asked 13 times.

⇒ * ENERGY EQUATION :→

$$TdS = dU + PdV$$

$$dU = TdS - PdV$$

(143)

$$\Rightarrow dU = \underbrace{C_v dT + T\left(\frac{\partial P}{\partial T}\right)_V dV}_{\text{from 1st T-ds Equation}} - PdV$$

$$\Rightarrow dU = \boxed{C_v} dT + \boxed{\left[T\left(\frac{\partial P}{\partial T}\right)_V - P\right]} dV \quad \text{——} ①$$

Now $U = f(T, V)$.

$$dU = \boxed{\left(\frac{\partial U}{\partial T}\right)_V} dT + \boxed{\left(\frac{\partial U}{\partial V}\right)_T} dV \quad \text{——} ②$$

Comparing equation ① & ②, we have.

$$C_v = \left(\frac{\partial U}{\partial T}\right)_V \; ; \quad \& \quad \left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P$$

Now for an ideal gas:

$$PV = mRT$$

$$P = \frac{mR}{V} \cdot T$$

for constant volume, this is of the form $y = cx$ ($\because \frac{mR}{V} = c$

$$\therefore \frac{dy}{dx} = c.$$

$$\therefore \left(\frac{\partial P}{\partial T}\right)_V = \frac{mR}{V}$$

Also, $\frac{mR}{V} = \frac{P}{T}$

$$\therefore \left(\frac{\partial P}{\partial T}\right)_V = \frac{mR}{V} = \frac{P}{T}$$

$$\left(\frac{\partial P}{\partial T}\right)_V = P/T$$

$$T\left(\frac{\partial P}{\partial T}\right)_V = P$$

$$\text{or } T\left(\frac{\partial P}{\partial T}\right)_V - P = 0 \quad \text{——} ③$$

Putting the value of ③ in eq^n ①:-

$$\boxed{dU = C_v\,dT + \left[T\left(\frac{\partial P}{\partial T}\right)_V - P\right]dV} \quad \text{——} Ⓗ$$

This is energy equation.

for ideal gas,

$$\left[T\left(\frac{\partial P}{\partial T}\right)_V - P\right] = 0.$$

$$\therefore \boxed{dU = C_v\,dT} \quad \text{——} ①$$

This is for an ideal gas.

for an ideal gas :-

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P = 0.$$

It means $\boxed{\left(\frac{\partial U}{\partial V}\right)_T = 0.}$ —— ([illegible]) (२).

i.e. ((The internal energy ~~charge does~~ of an ideal gas does not change even if the volume changes.

It means internal energy is independent of volume for an ideal gas.))

With Respect to (α):

For an ideal gas under isothermal conditions, there is no change of internal energy with respect to volume. i.e. internal energy of an ideal gas is independent of volume.

$$U = f(T, P, V)$$

$$T = \phi(U, P, V)$$

from 2nd theorem, we have:-

$$\left(\frac{\partial U}{\partial P}\right)_T \left(\frac{\partial P}{\partial V}\right)_T \left(\frac{\partial V}{\partial U}\right)_T = 1$$

$$\left(\frac{\partial U}{\partial V}\right)_T = \left(\frac{\partial U}{\partial P}\right)_T \left(\frac{\partial P}{\partial V}\right)_T$$

for an ideal gas, $\left(\frac{\partial U}{\partial V}\right)_T = 0$.

$$0 = \left(\frac{\partial U}{\partial P}\right)_T \cdot \left(\frac{\partial P}{\partial V}\right)_T$$

↳ This is always negative.

$$\therefore \boxed{\left(\frac{\partial U}{\partial P}\right)_T = 0} \quad \text{——} (\beta)$$

With respect to (β):

This shows that the internal energy of an ideal gas does not vary with respect to pressure under isothermal conditions. Therefore, internal energy of an ideal gas is a function of temperature.

⇒ * JOULE THOMSON COEFFICIENT (μ) :-

Joule Thomson co-efficient shows variation of temperature with respect to pressure under throttling or isenthalpic conditions.

$$\boxed{\mu = \left(\frac{\partial T}{\partial P}\right)_h} \text{ —— Ⓙ}$$

146

⇒ *

$$T\,ds = dh - v\,dP$$

$$dh = T\,ds + v\,dP$$

$$\Rightarrow dh = C_p\,dT - T\left(\frac{\partial v}{\partial T}\right)_P dP + v\,dP$$

(from 2nd Tds Equation)

$$\Rightarrow dh = C_p\,dT - \left[T\left(\frac{\partial v}{\partial T}\right)_P - v\right] dP \text{ —— Ⓣ}$$

for an ideal gas:-

$$PV = mRT$$

$$V = \frac{mR}{P}\cdot T$$

This is of the form $y = cx$ $(\because P = c)$.

$$\therefore \left(\frac{\partial v}{\partial T}\right)_P = \frac{mR}{P}$$

But, $\frac{mR}{P} = \frac{V}{T}$.

$$\therefore \left(\frac{\partial v}{\partial T}\right)_P = \frac{mR}{P} = \frac{V}{T}$$

or, $T\left(\frac{\partial v}{\partial T}\right)_P - V = 0$

$$\therefore\ dh = C_p\,dT - \left[T\left(\frac{\partial V}{\partial T}\right)_P - V\right] dP$$

$$\Rightarrow dh = C_p\,dT - 0 \qquad (\text{for ideal gas})$$

$$\Rightarrow \boxed{dh = C_p\,dT} \ \text{———} \ (k)$$

for an ideal gas, $dh = C_p\,dT$.

for throttling, $h =$ const, $dh = 0$.

$$\therefore\ 0 = C_p\,dT - \left[T\left(\frac{\partial V}{\partial T}\right)_P - V\right] dP$$

$$\text{or,}\ C_p\,dT = \left[T\left(\frac{\partial V}{\partial T}\right)_P - V\right] dP$$

$$\frac{dT}{dP} = \frac{1}{C_p}\left[T\left(\frac{\partial V}{\partial T}\right)_P - V\right]$$

$$\left(\frac{\partial T}{\partial P}\right)_h = \frac{1}{C_p}\left[T\left(\frac{\partial V}{\partial T}\right)_P - V\right]$$

$$\Rightarrow \mu = \frac{1}{C_p}\left[T\left(\frac{\partial V}{\partial T}\right)_P - V\right].$$

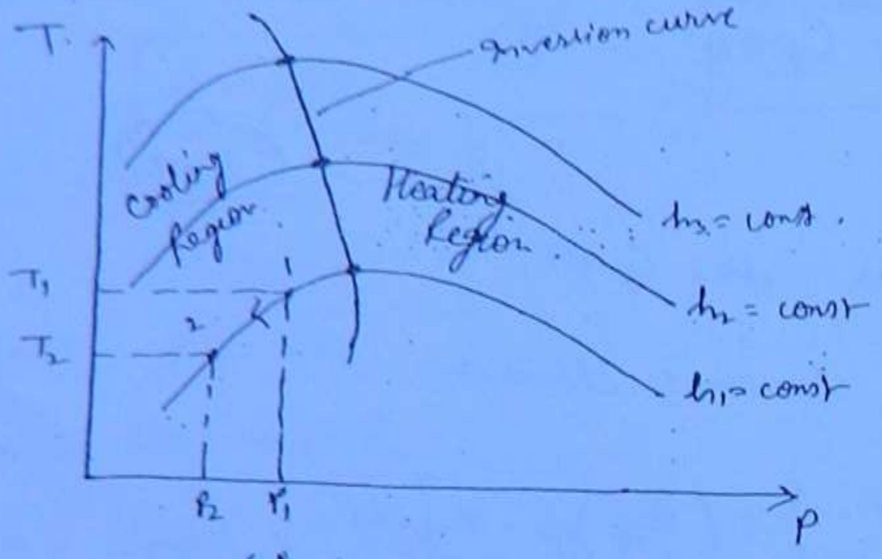
for an ideal gas,

$$T\left(\frac{\partial V}{\partial T}\right)_P - V = \underline{0}$$

$$\therefore\ \boxed{\mu = 0} \quad (\text{for an ideal gas}) \ \text{———} \ (L)$$

Joule-Thomson co-efficient for an ideal gas is zero.

For an ideal gas, as enthalpy is a function of temperature, under throttling conditions, as enthalpy remains constant, temperature also remains constant.



Slope → $\left(\frac{\partial T}{\partial P}\right)_h$

$\mu = \left(\frac{\partial T}{\partial P}\right)_h$ = Slope of isenthalpic curves on T-P diagram.

For cooling Region, μ = ⊕ve.
For heating Region, μ = ⊖ve.

Joule Thomson co-efficient is positive or ~~negat~~ slope of isenthalpic curves on T-P diagram is positive in cooling region. Similarly Joule Thomson co-efficient or slope of isenthalpic curves is negative in heating region.

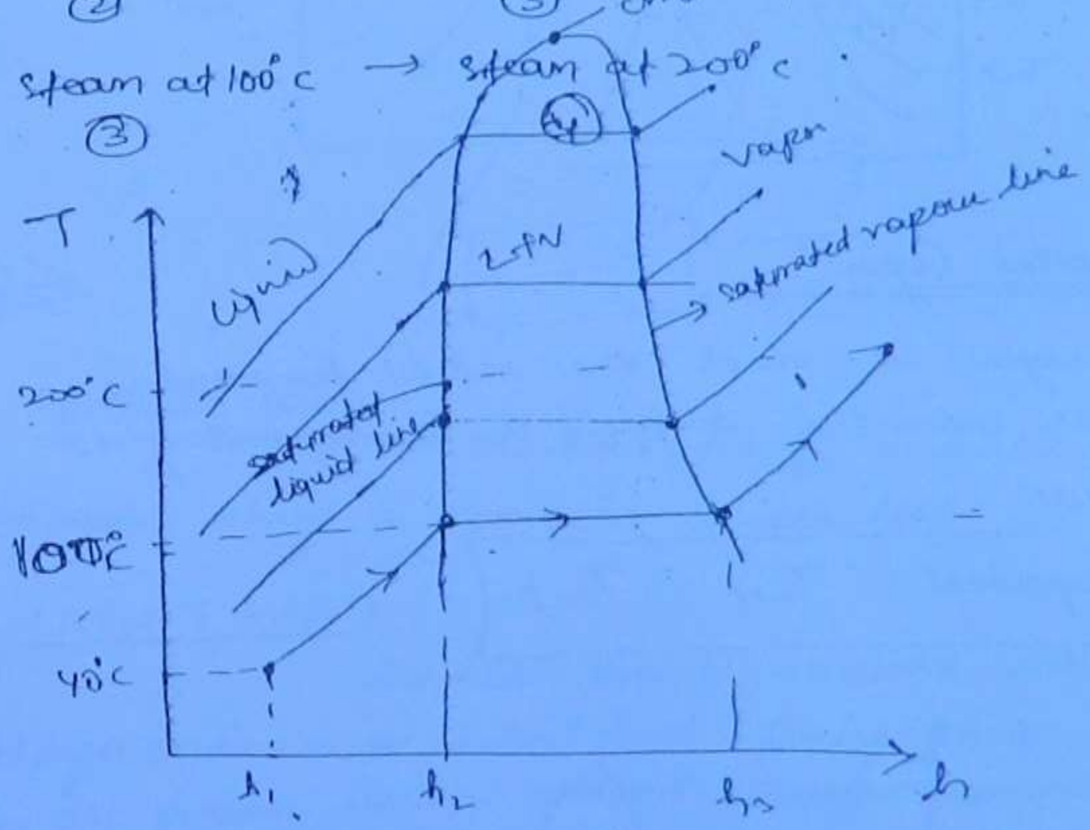
PROPERTIES OF PURE SUBSTANCES

H_2O at 40°C ⟶ steam at 200°C
when pressure is atmospheric.

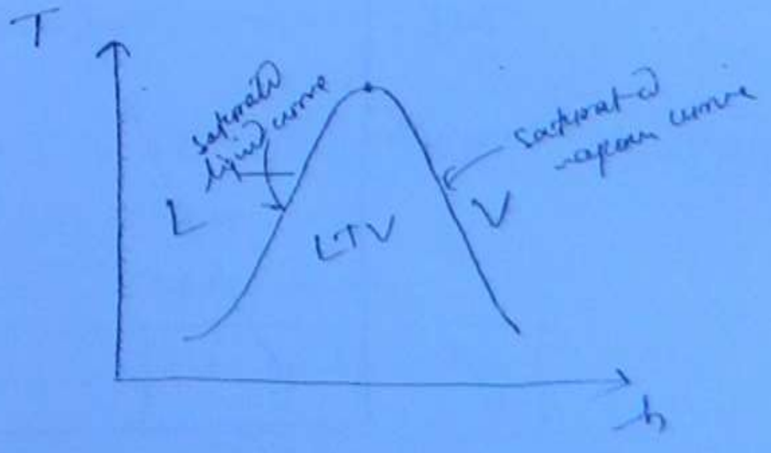
I. H_2O at 40°C ⟶ H_2O at 100°C
① ②

II. H_2O at 100°C ⟶ ~~H_2O~~ steam at 100°C
② ③

III. steam at 100°C ⟶ steam at 200°C
③



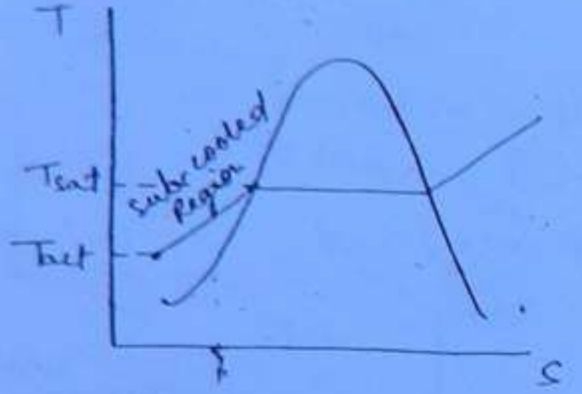
Critical point is the point at which saturated liquid and saturated vapour meets.



⇒* Sub-cooled or under cooled Region :-

It is a region in which the actual temperature is less than saturation temperature corresponding to that pressure.

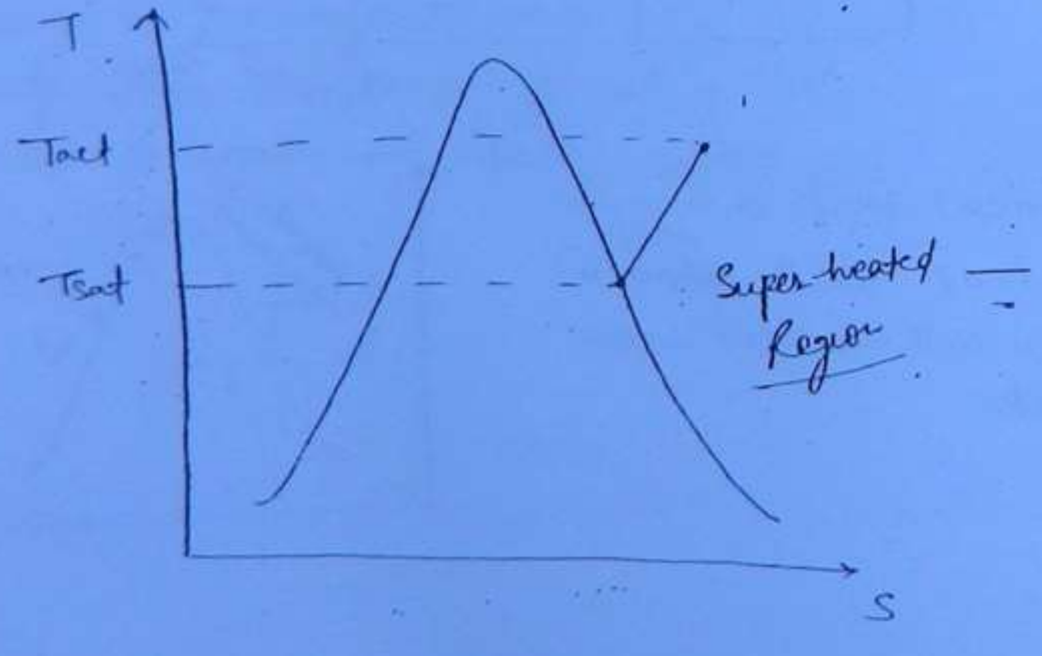
$$\boxed{\text{Degree of subcooling} = T_{saturation} - T_{actual}}$$



⇒* Super-heated Region :-

It is a region in which the actual temperature greater than saturation temperature corresponding to that pressure.

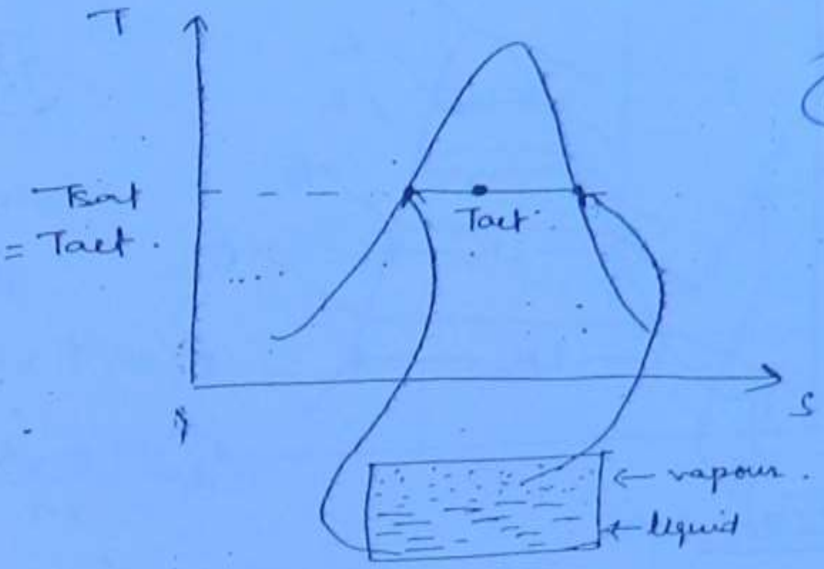
$$\boxed{\text{Degree of superheat} = T_{act} - T_{sat}}$$



⇒* Wet Region :-

It is a region in which both liquid and vapour exist in equilibrium. For wet region, the actual temperature is equal to saturation temperature i.e. ($T_{act} = T_{sat}$).

(157)

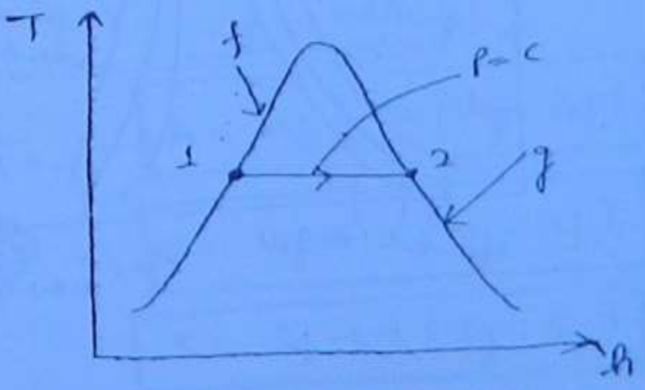


⇒* SENSIBYE HEAT :-

The heat transfer associated with temperature change is known as sensible heat.

⇒* LATENT HEAT :-

The heat transfer associated with phase change is known as latent heat. During phase change as the pressure remains constant, therefore latent heat is equal to change in enthalpy.



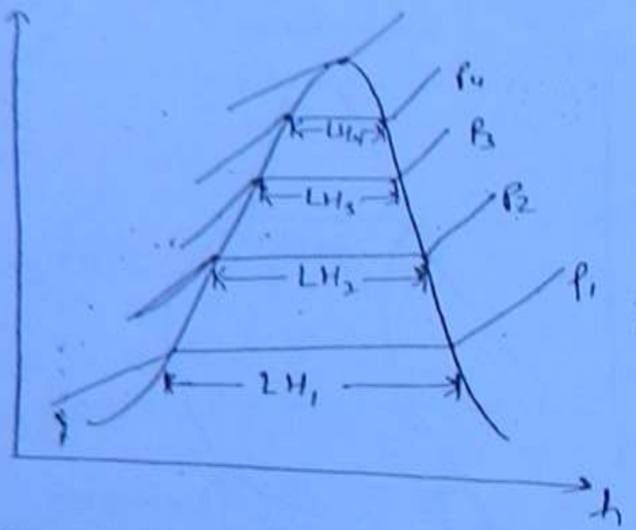
$$LH = Q_{12} = Q_p = dH$$

$$LH = dh$$

$$LH = h_2 - h_1$$

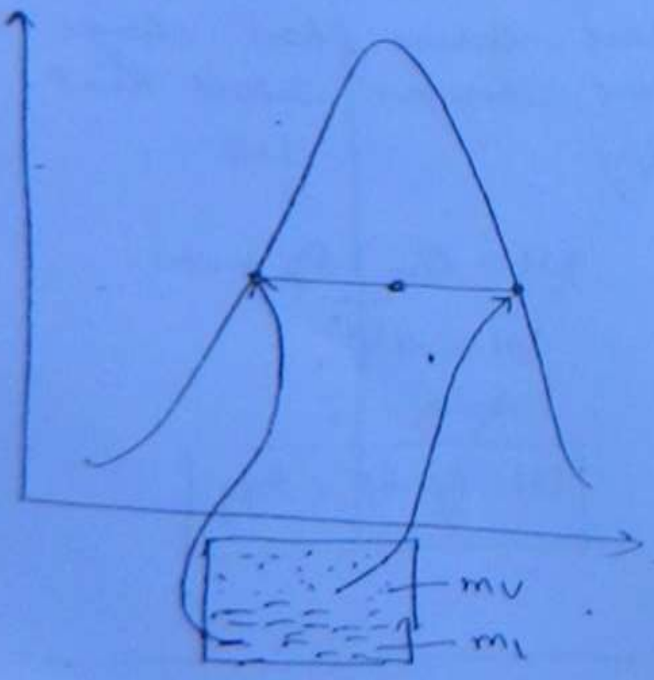
$$\boxed{LH = h_g - h_f = h_{fg}}$$

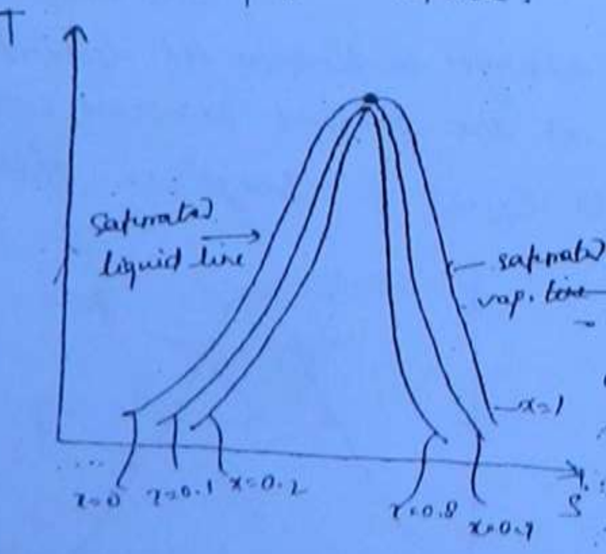
Note:- With increase in pressure, the latent heat of vaporization decreases and at critical point latent heat of vaporization is equal to zero.



⇒ * DRYNESS FRACTION :→

It is defined as the ratio of mass of vapour to the total mass of the mixture. The dryness fraction along saturated liquid curve = 0 and the dryness fraction along saturated vapour curve is equal to 1. The dryness fraction is also known as quality of mixture.





$$\boxed{x = \frac{m_v}{m_v + m_l}}$$

⇒* SPECIFIC VOLUME OF MIXTURE :- ⑮③

$v = V/m$

$V = vm.$

$m = m_v + m_l$

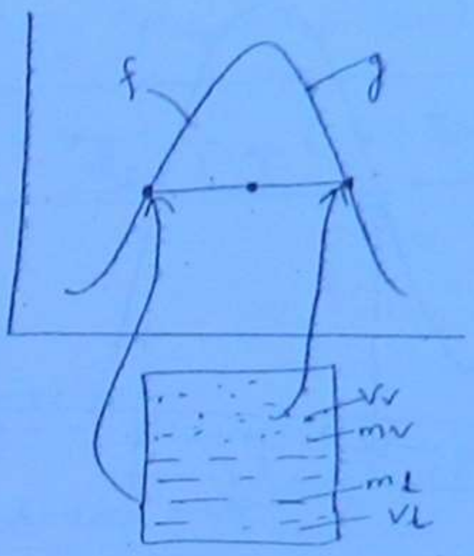
$V = V_v + V_l$

$mv = m_v v_v + m_l v_l$

$$v = \frac{m_v v_v + m_l v_l}{m}$$

$$v = \frac{m_v}{m} v_v + \frac{m_l}{m} v_l$$

$$v = \frac{m_v}{m_v + m_l} v_v + \frac{m_l}{m_v + m_l} v_l$$



$$x = \frac{m_v}{m_v + m_l}.$$

$$1 - x \Rightarrow 1 - \frac{m_v}{m_v + m_l}.$$

$$1 - x = \frac{m_l}{m_v + m_l}.$$

$v = x v_v + (1 - x) v_l$

$v = x v_v + v_l - x v_l$

$v = v_l + x(v_v - v_l)$

$$\boxed{v = v_f + x(v_g - v_f)}$$

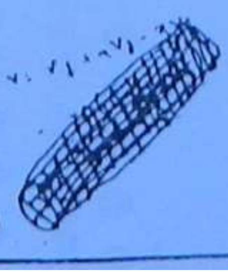
Similarly

$$\boxed{h = h_f + x(h_g - h_f)}$$

$$\boxed{u = u_f + x(u_g - u_f)}$$

$$\boxed{s = s_f + x(s_g - s_f)}$$

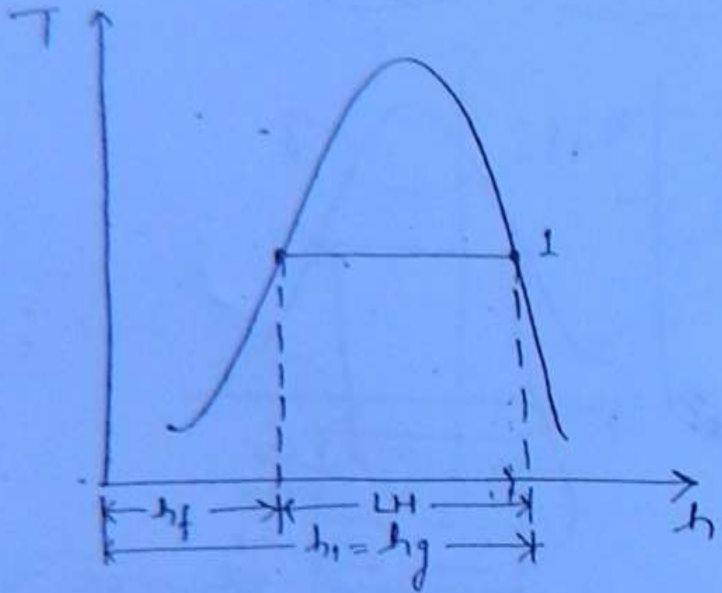
These equations must be used when the point is in wet region.



-: ENTHALPY AT VARIOUS POINTS :- (V.V.I)

CASE-I:- when the point is on saturated vapour curve.

154

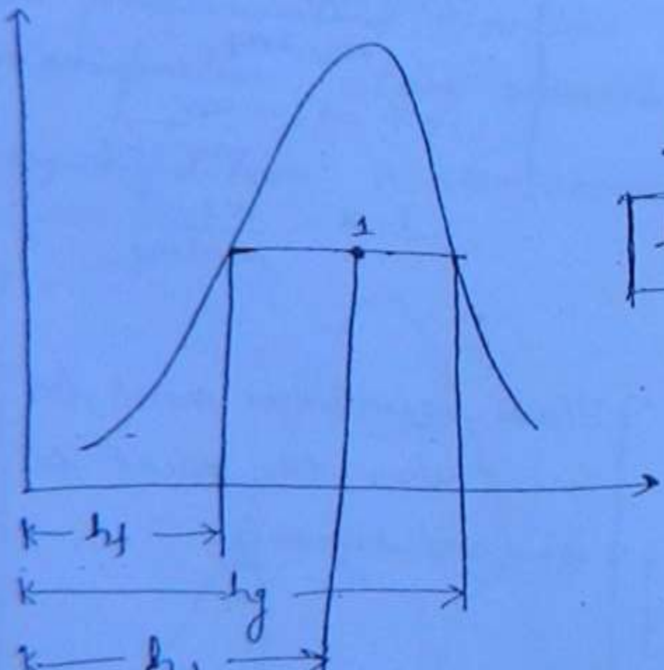


$$h_g = h_1 - h_f + LH$$

$$h_g = h_f + LH$$

~~LH =~~ $\boxed{h_g - h_f = LH}$

Case-II: when the point is in wet region.

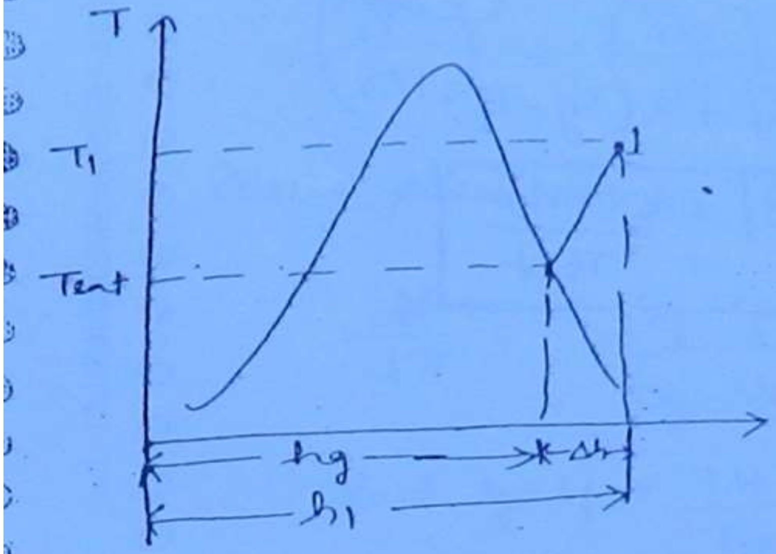


$$h_1 = h_f + x(h_g - h_f)$$

$$\boxed{h_1 = h_f + x(LH)}$$

$$\because (h_g - h_f) = h_{fg} = LH.$$

Case - III : when the point is in superheated region.



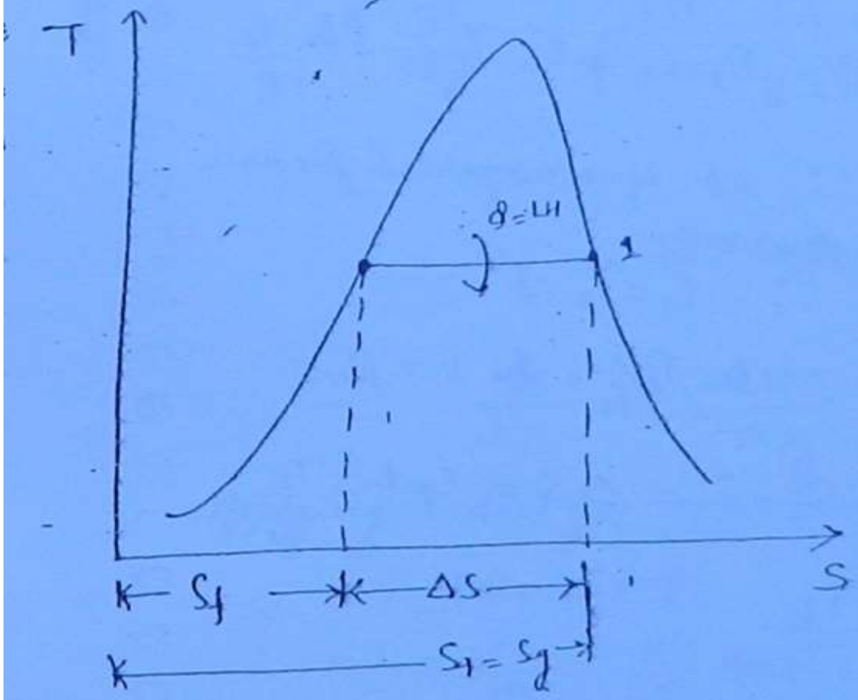
$$h_1 = h_g + \Delta h$$

$$\Delta h = C_{p\,vap.}(T_1 - T_{sat})$$

$$\boxed{h_1 = h_g + C_{p\,vap.}(T_1 - T_{sat})}$$

⇒ * ENTROPY AT VARIOUS POINTS :-

Case-I : when the point is on saturated vapour curve.



$$S_1 = S_f + \Delta S$$

$$dS = \frac{dQ}{T}$$

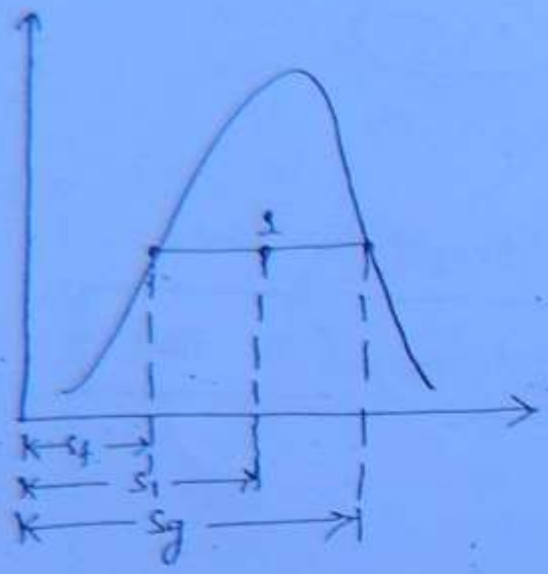
$$\Delta S = \frac{LH}{T_{sat}}$$

$$S_1 = S_f + \frac{LH}{T_{sat}}$$

$$S_g = S_f + \frac{LH}{T_{sat}}$$

$$\boxed{S_g - S_f = \frac{LH}{T_{sat}}}$$

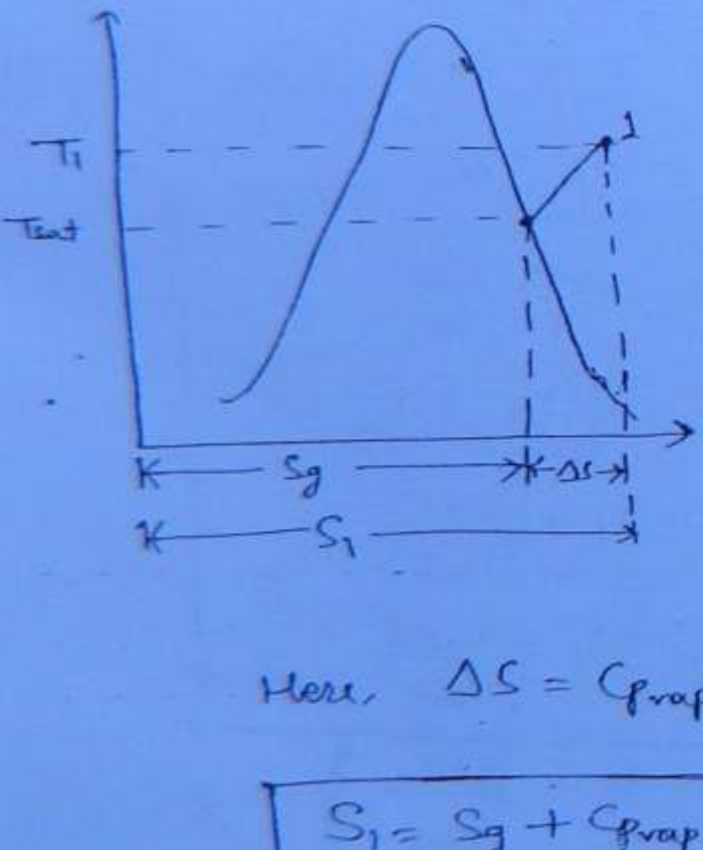
Case-II: When the point is in wet region.



$$S_1 = S_f + x(S_g - S_f)$$

$$\boxed{S_1 = S_f + \frac{x(LH)}{T_{sat}}}$$

Case-III: When the point is in super heated region.



$$S_1 = S_g + \Delta S$$

$$S_2 - S_1 = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

∵ It is a constant pressure process.

∴ $P_1 = P_2$.

∴ $\ln P_2/P_1 = \ln 1 = 0$.

∴ $S_2 - S_1 = \Delta S = C_p \ln \frac{T_2}{T_1}$

Here, $\Delta S = C_{p\,vap} \ln \frac{T_1}{T_{sat}}$

$$\boxed{S_1 = S_g + C_{p\,vap.} \ln \frac{T_1}{T_{sat}}}$$

⟹ * VVI CLAUSIUS CLAPEYRON EQUATION :→

$$\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial s}{\partial v}\right)_T$$ (157)

During phase change P & T are independent of volume.

$$\therefore \frac{dP}{dT} = \frac{s_g - s_f}{v_g - v_f}$$

But $s_g - s_f = \frac{LH}{T_{sat}}$.

$$\therefore \boxed{\frac{dP}{dT} = \frac{LH}{T_{sat}(v_g - v_f)}} \longrightarrow \text{Clapeyron Equation}$$

05/08/2011

$$\frac{dP}{dT} = \frac{LH}{T_{sat}(v_g - v_f)}$$

$\because v_g >> v_f$

$\therefore v_f$ can be neglected.

Thus $$\frac{dP}{dT} = \frac{LH}{T v_g} \quad \text{——} ①$$

Applying ideal gas equation,

$$PV = mRT$$

$$\Rightarrow \frac{V}{m} = \frac{RT}{P}$$

$$\text{or } v = \frac{RT}{P} \quad \text{——} ②$$

Putting the value of ① in eqⁿ ①, we have.

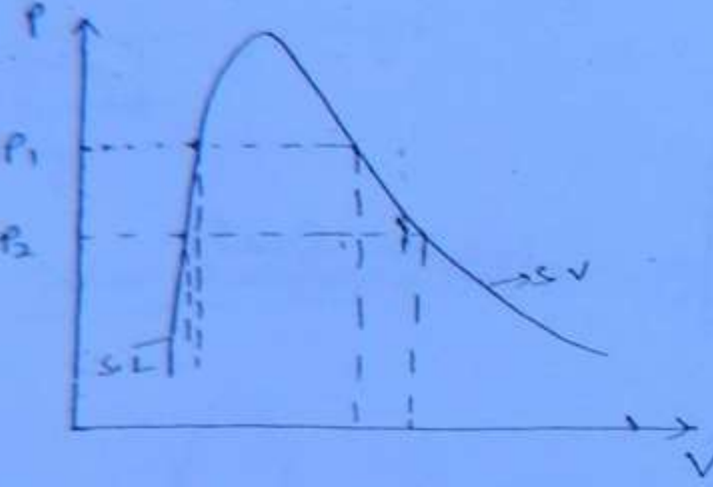
$$\frac{dP}{dT} = \frac{LH}{T \times \frac{RT}{P}}$$

$$\text{or, } \boxed{\frac{dP}{dT} = \frac{P(LH)}{RT^2}}$$

*** V.V.I

— This is known as Clausius - Clapeyron Equation.

⇒* P-V DIAGRAM!→



~~* MOLLIER DIAGRAM!→~~

ote:-

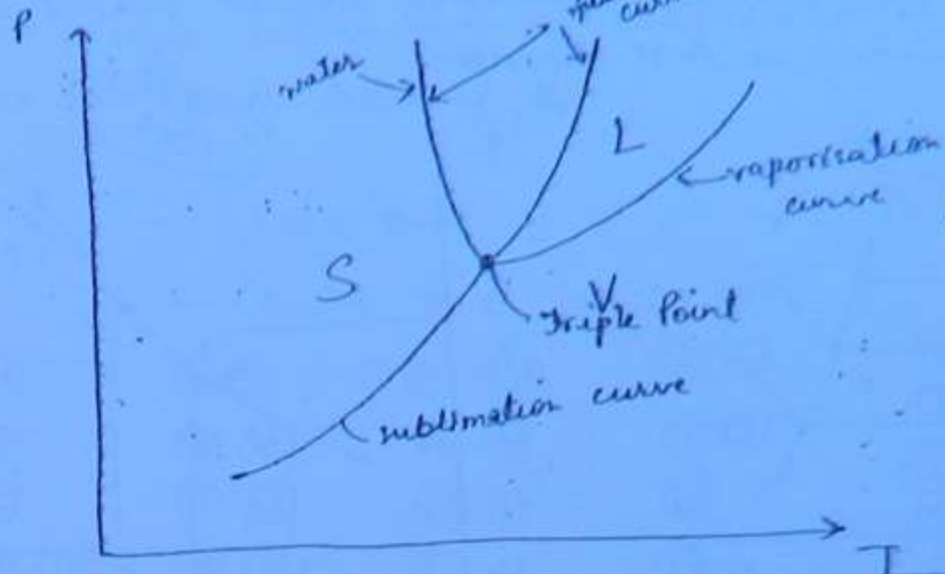
In subcooled region and superheated region, both temperature and pressure can be varied independently and hence, the degree of freedom in these regions is equal to two (2).

But in wet region, constant pressure and constant temperature lines are same and hence, the degree of freedom in wet region is equal to one (1).

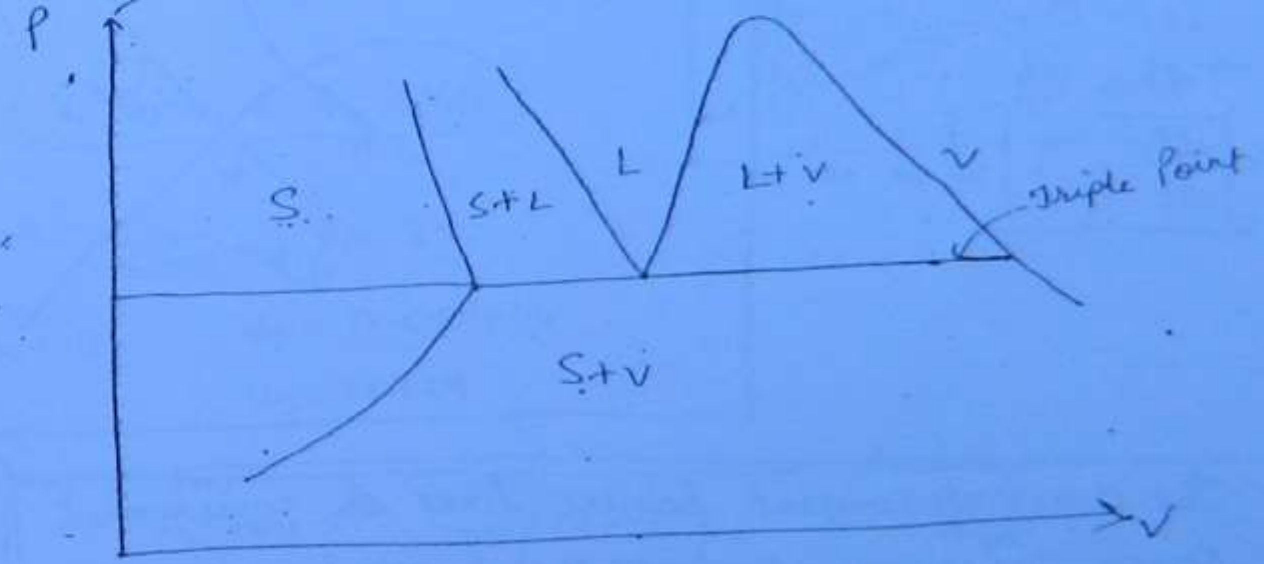
: → L → Melting
→ S → freezing
→ V → Vaporization
V → L → condensation
S → V → Sublimation

General Substances	water
S-L → Melts → Expands	S-L → Melts → Contract
L-S → freeze → Contracts	L-S → freeze - Expands
	because of element of hydrogen bonding and cage like structure.

⇒* Solid - Liquid and Vapour Curve with triple point :-



<u>Note</u>:- The slope of fusion curve on P–T diagram is positive for general substances and negative for water.

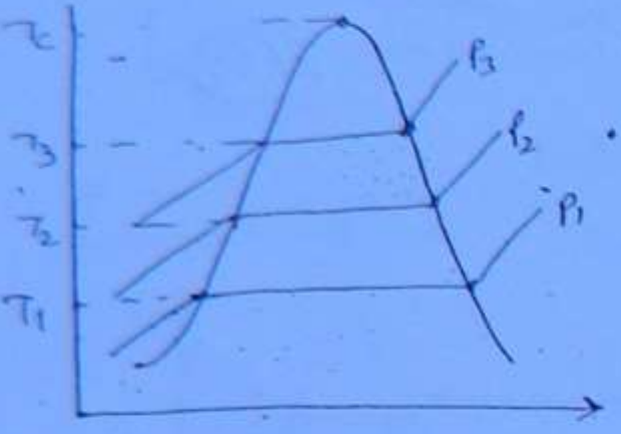


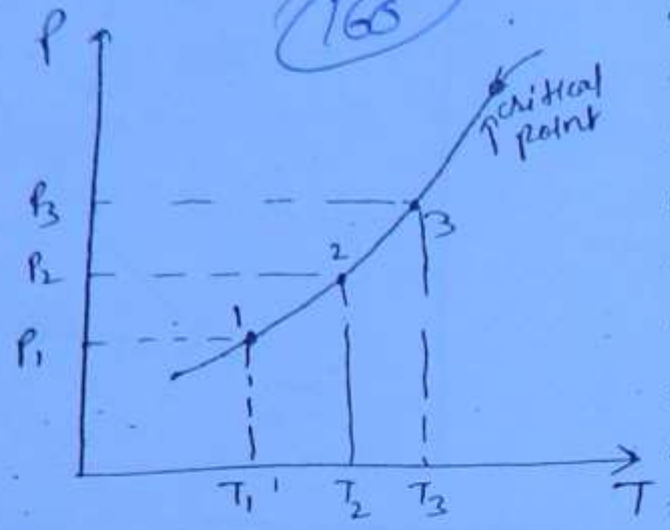
Triple Point data for water

$P_{tp} = 0.006113$ bar

$t_{tp} = 0.01°C$.

According to Gibb's Phase Rule, the degree of freedom at triple point is 0 i.e. no intensive parameters can be varied at triple point. Therefore, it is a point on P-T diagram, but extensive properties can be varied and hence it is a line on P-V diagram.





⇒* MOLLIER DIAGRAM :→

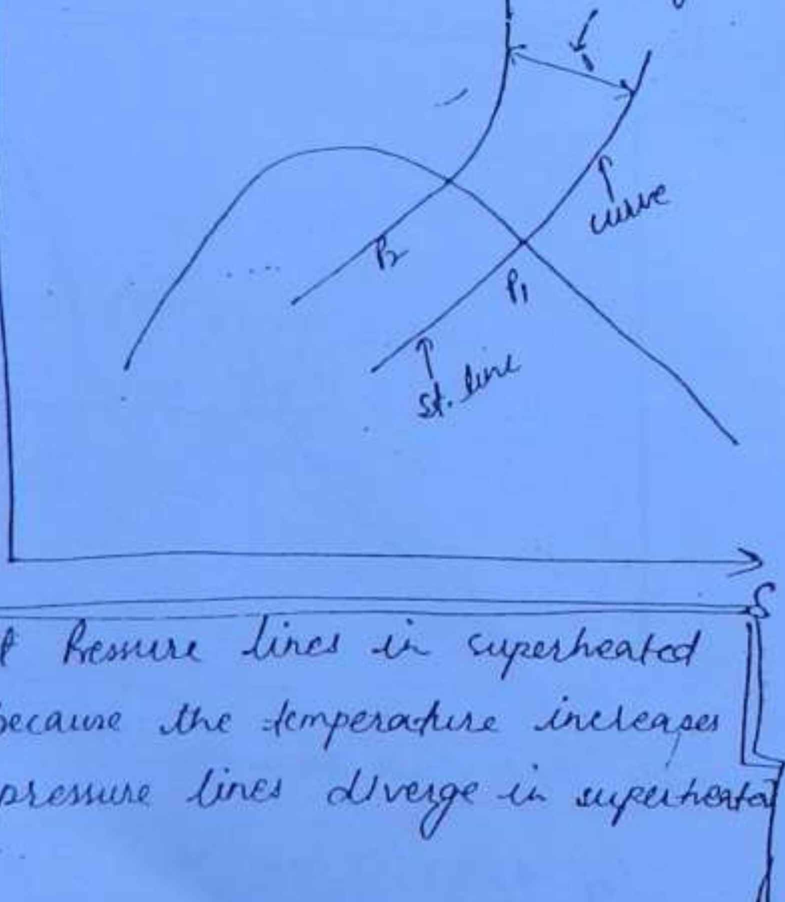
$Tds = dh - vdP$

$P = \text{const}$

$dP = 0$

$Tds = dh$

or, $\boxed{\frac{dh}{ds} = T}$



Note:- The slope of constant Pressure lines in superheated region increases because the temperature increases and hence constant pressure lines diverge in superheated region.

⇒ * REFERENCE STATE IN STEAM TABLE :→

Internal energy and entropy of saturated water at triple point is arbitrarily taken as zero.

(161)

Work Book

Chapter -6

① ⓓ
② ⓐ
③ ⓒ
④ ⓐ, ⓑ
small, capital
⑤ ⓒ

⑥ ⓒ
⑦ ⓑ
⑧ ⓓ
⑨ ⓑ
⑩ ⓒ

⑪ ⓐ
⑫ ⓒ
⑬ ⓑ
⑭ ⓑ
⑮ ⓒ

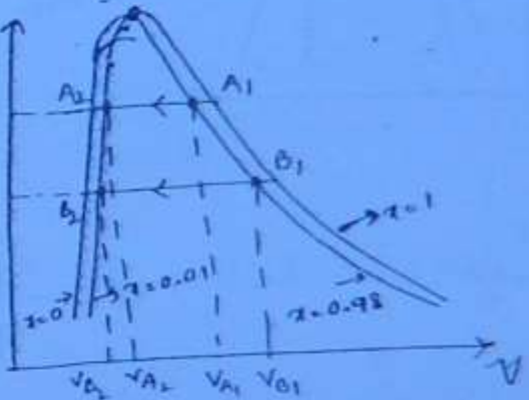
⑯ ⓑ
⑰ ⓓ
⑱
⑲
⑳

㉑
㉒
㉓
㉔
㉕
㉖

①

B₁
200 kPa
$x = 0.01$

A₁
300 kPa
$x = 0.98$



ⓓ $\left(V_{B_1} > V_{A_1} \;\&\; V_{A_2} > V_{B_2}\right)$

② ~~$x = \frac{m_v}{m_v+m_l}$~~ $V_v = 8V_L$

$x = \frac{m_v}{m_v + m_l}$ $v_f = 0.001044$

$v_g = 1.6729$

$$\therefore \quad x = \frac{m_v}{m_v+m_l} = \frac{m_v}{m_v\left(1+\frac{m_l}{m_v}\right)} = \frac{1}{1+\frac{m_l}{m_v}}$$

$$v = V/m \quad \text{or} \quad m = V/v$$

$$\therefore \; m_v = \frac{V_v}{v_v} = \frac{8V_l}{v_v}$$

$$m_l = \frac{V_l}{v_l}$$

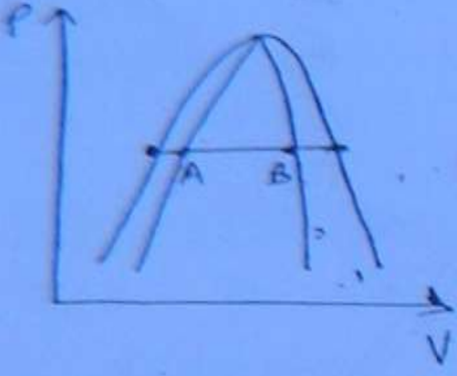
$$\therefore x = \frac{1}{1+\frac{\frac{V_l}{v_l}}{\frac{8V_l}{v_v}}} \Rightarrow x = \frac{1}{1+\frac{V_l}{v_l}\times\frac{v_v}{8V_l}}$$

162

$$= \frac{1}{1+\frac{1}{8}\left(\frac{1.6729}{0.001044}\right)}$$

$$\therefore \boxed{x = 0.0049 \simeq 0.005} \quad (a)$$

③



$$U_A = U_B$$

$$m_A u_A = m_B u_B$$

$$\therefore \frac{m_A}{m_B} = \frac{u_B}{u_A} = \frac{u_f + x_B(u_g - u_f)}{u_f + x_A(u_g - u_f)}$$

$$= \frac{\cancel{u_f} + u_g - \cancel{u_f}}{u_f}$$

$$= \frac{u_g}{u_f} = \frac{2583.6}{761.7} = \underline{\underline{3.4}} \quad (c)$$

⑤

⑦ $\rho = 36.12\ kg/m^3$

$T = -30°C = 243\ K.$

$\rho_f = 1377\ kg/m^3, \quad \rho_g = 7.379\ kg/m^3.$

$v = v_f + x(v_g - v_f)$

$v = V/m = \frac{1}{m/V} = 1/\rho.$

$\rho = m/V$

$$\therefore \frac{1}{\rho} = \frac{1}{\rho_f} + x\left(\frac{1}{\rho_g} - \frac{1}{\rho_f}\right)$$

$$\Rightarrow \frac{1}{36.12} = \frac{1}{1377} + x\left(\frac{1}{7.379} - \frac{1}{1377}\right)$$

$\therefore \boxed{x = 0.2}$ ⓑ

⑧ $\underline{Q = h_2 - h_1}$

Given: $x = 0.4$, $V = 1\ m^3$, $P = 200\ kPa$, $h_2 = 125.7\ kJ/kg$.

$h_1 = h_f + x(h_g - h_f)$

$= 504.7 + 0.4(2706.7 - 504.7)$

$\therefore h_1 = 1385.5\ kJ/kg.$

$\therefore Q = h_2 - h_1 = (125.7 - 1385.5) = -1259.8\ kJ/kg$

(-)ve sign mean heat rejection.

Now, $v = V/m \Rightarrow m = V/v$

$v = v_f + x(v_g - v_f)$

$= 0.001061 + 0.4(0.8857 - 0.001061)$

$\therefore v = 0.355$

$\therefore m = V/v = \frac{1}{0.355} = 2.8$ kg.

$\therefore Q = -1259.8 \times 2.8$

$= -3550$ kj ⓓ

(164)

⑨ $T = 99.63^\circ C$

~~$S = S_f + x(S_g - S_f)$~~

$(S_g - S_f) = \frac{LH}{T_{sat}}$

$\Rightarrow (S_g - S_f) = \frac{(h_g - h_f)}{T_{sat}} = \frac{2675.5 - 417.5}{372.63}$

$\therefore (S_g - S_f) = 6.9596$

$\therefore S_g = S_f + 6.0596 = 1.3026 + 6.0596$

$\therefore \boxed{S_g = 7.362 \text{ kJ/kg K}}$ ⓑ

⑩ $V = 20 m^3$

$x = 1$, $P_1 = 1$ MPa. $P_2 = 0.1$ MPa.

$T = 300$ K

$Q = u_2 - u_1$

The system undergoes constant volume process and hence heat transfer is equal to change in internal energy.

$u_1 = u_f - x(u_g - u_f)$

$=$

$u_2 =$

(14) Adiabatic ($đQ$) = 0

Rev.

$ds = \frac{dQ}{T}$

$dS = 0$

$S = \text{Const.}$

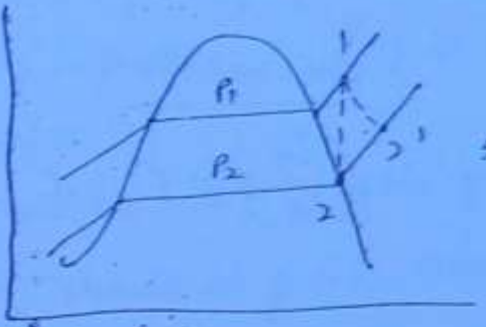
Irrev.

(165)

$dS = \left(\frac{đQ}{T}\right)_{irr.} + (\delta s)_{gen.}$

$dS = (\delta S)_{gen}$; $(\delta S) > 0$

$\boxed{dS > 0}$



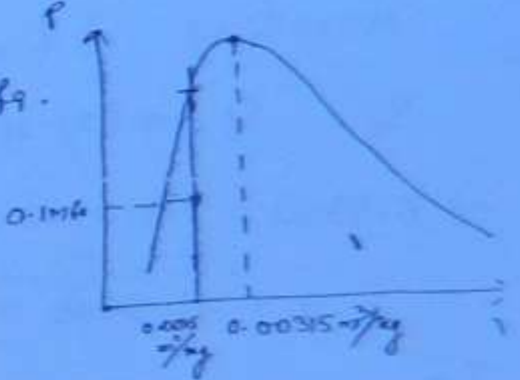
(18) $v_c = 0.003155 \, m^3/kg$.

$V = 0.025 \, m^3$, $P = 0.1 MPa = 100 kPa$.

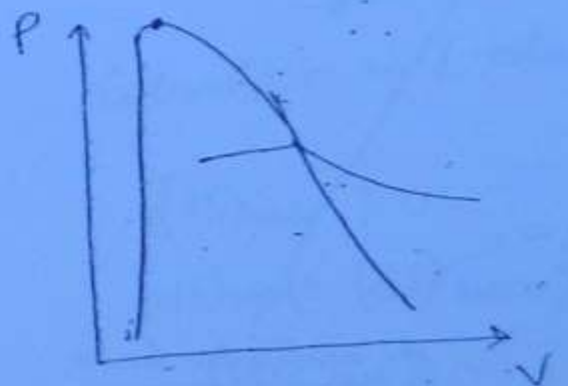
$m = 10 \, kg$.

$v = V/m = 0.0025 \, m^3/kg$

As the in constant volume heating, the line is moving towards liquid line. Hence, liquid level will rise.



(19)

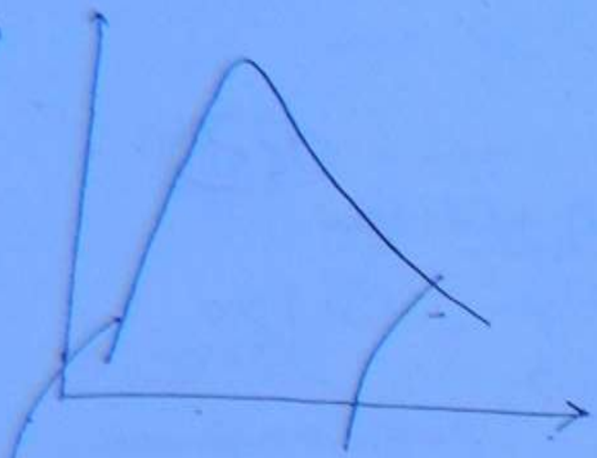


$PV = mRT \rightarrow C$

$\boxed{PV = C}$

(21)

(22)



$\frac{dP}{dv} = 0$ $\frac{d^2P}{dv^2} = 0$

$\frac{d^3P}{dv^3} < 0.$

from Mathematics.

for max^m:

$\left(\frac{dy}{dx}\right) = 0$ & $\frac{d^2y}{dx^2} < 0.$

for min^m:

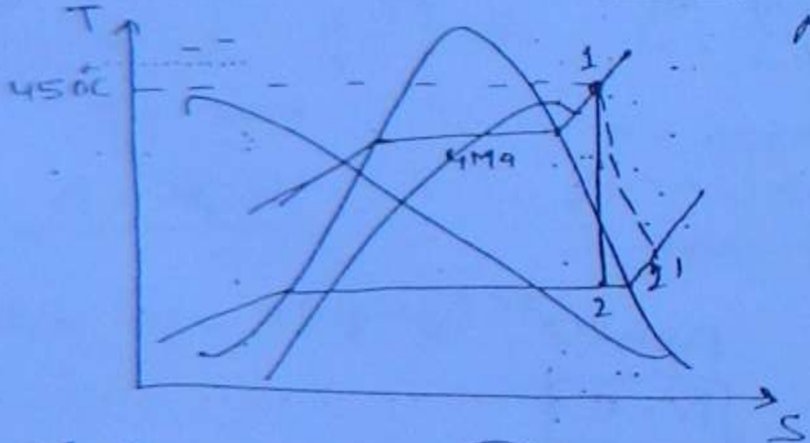
$\left(\frac{dy}{dx}\right) = 0$ & $\frac{d^2y}{dx^2} > 0.$

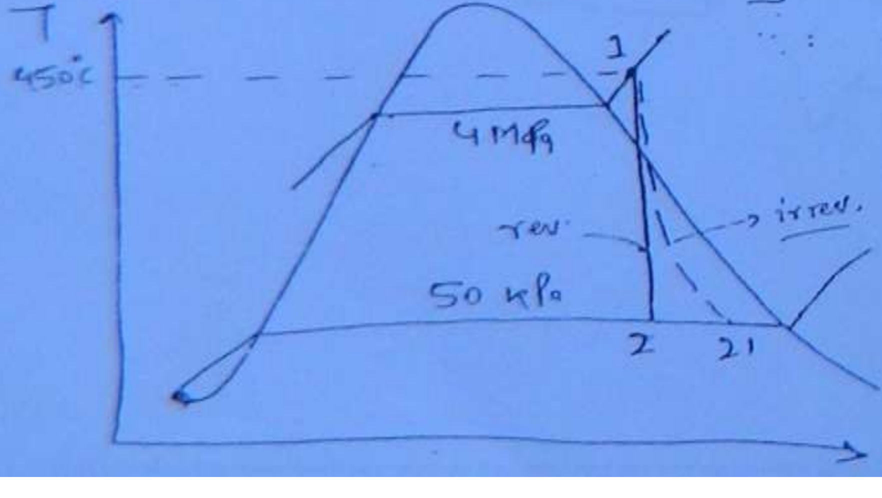
(23)

$s_2 = s_f + x(s_g - s_f)$

$\Rightarrow 6.9362 = 1.091 + x(7.5939 - 1.091)$

$\therefore x = 0.8988 \simeq \underline{\underline{0.9}}.$





for adiabatic process,
only two conditions
are possible
S = constant or
S increasing.
x is minimum for 2.

㉕

1	2	3	4	5	6	7	8
t°C	P(kPa)	u_f	s_f	h_f	s_g	u_g	h_g
-20	190	88.76	0.3657	89.05	5.6155	1299.5	1418
40	1554.9	368.74	1.3574	371.43	4.8662	1341	1470.2

(167)

$h = u + Pv$

$h_f = u_f + Pv_f$

v_f is small, ∴ Pv_f is small.

$h_f \simeq u_f$

But h_f will be slightly greater than u_f.

u_g & h_g should be greater than u_f and h_f.

∴ Column ④ and ⑥ are not u_g & h_g. So they are s_f and s_g.

Also, s_g is greater than s_f

∴ Column ⑥ is s_g and column ④ is s_f.

Again, $h_g = u_g + P_g v_g$

∴ $h_g > u_g$.

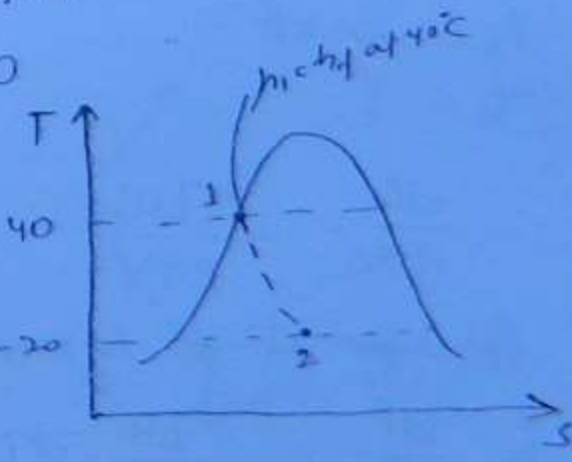
Column ⑦ is u_g & column ⑧ is h_g.

㉖

$(dS)_{univ} > 0$ ∵ $\overset{\text{throttling}}{(\because \text{adiabatic})}$

$(dS)_{sys} + \overset{\nearrow 0}{(dS)_{surr}} > 0$

$(dS)_{sys} > 0$



$h_1 = h_2 = 371.43$

$h_2 = h_f + x(h_g - h_f)$

$371.43 = 89.05 + x(1418 - 89.05)$

$\therefore x = \underline{\underline{0.212}}$

Work Book

Chapter - 7

① - (d) ⑥ - (d) ⑪ - (b) ⑯ - (a)

② - (a) & (b) ⑦ - (b) ⑫ - (c) ⑰ - (b)

③ - (b) ⑧ - (a) ⑬ - (d) ⑱ - (a)

④ - (b) ⑨ - ~~(a)~~ (b) ⑭ - (d) ⑲ - (a)

⑤ - (c) ⑩ - (c) ⑮ ....

① $\left(\frac{dP}{dT}\right) = 17.69$ kPa/K.

$v_f = 0.00081\,57$ m³/kg, $v_g = 0.0358$ m³/kg.

from Clapeyron eqn :-

$$\frac{dP}{dT} = \frac{LH}{T(v_g - v_f)}$$

$$LH = 17.69 \times 293\,(0.0358 - 0.00081\,57)$$

$$= \underline{\underline{181.4}} \quad (d)$$

② $T_1 = 273 + 76 = 349$ K.

$P_1 = 101$ kPa.

$LH = 195$ kJ/kg.

$R = 0.055$ kJ/kg K.

$P_2 = 202$ kPa

$T_2 = ?$

$$\frac{dP}{dT} = \frac{P(LH)}{RT^2}$$

Assuming LH to be constant,

$$\frac{dP}{P} = \frac{LH}{R} \cdot \frac{dT}{T^2}$$

Integrating above equation, we get.

(169)

$$\int_{P_1}^{P_2} \frac{dP}{P} = \int_{T_1}^{T_2} \frac{LH}{R} \cdot \frac{dT}{T^2}$$

$$\ln \frac{P_2}{P_1} = \frac{LH}{R} \left[-\frac{1}{T} \right]_{T_1}^{T_2}$$

$$\ln \frac{P_2}{P_1} = \frac{-LH}{R} \left[\frac{1}{T_2} - \frac{1}{T_1} \right]$$

$$\Rightarrow \ln \frac{202}{101} = -\frac{195}{0.055} \left[\frac{1}{T_2} - \frac{1}{349} \right].$$

$$\therefore \boxed{T_2 = 374.5k} \text{ (Ans)}$$

④ G-F

$(H-TS) - (U-TS)$

$H-\cancel{TS}-U+\cancel{TS}$

$\therefore H-U = PV$

But, for ideal gas, $PV = mRT$

$\therefore H-U = mRT$

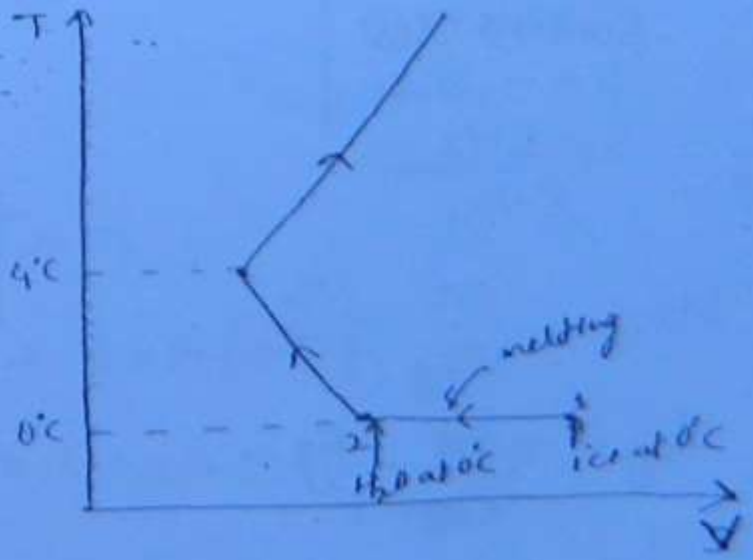
$\therefore$ H-U depends on T. (b)

⑤ $\left(\frac{\partial S}{\partial P}\right)_T = \left(-\frac{\partial V}{\partial T}\right)_P$

$\left(\frac{\partial V}{\partial T}\right)_P$ from 0°C to 4°C is $\ominus$ve.

But $\left(\frac{\partial S}{\partial P}\right)_T = \left(-\frac{\partial V}{\partial T}\right)_P$

$\therefore \left(\frac{\partial S}{\partial P}\right)_T$ for 0°C to 4°C is positive.



$\left(\frac{\partial V}{\partial T}\right)_P$ above 4°C is +ve.

(170)

but $\left(\frac{\partial S}{\partial P}\right)_T = \left(-\frac{\partial V}{\partial T}\right)_P$

$\therefore \left(\frac{\partial S}{\partial P}\right)_T$ is (−)ve for temp above 4°C

$\therefore$ Answer is (A)

$\left(\frac{\partial S}{\partial P}\right)_T > 0$ at 3°C and < 0 at 5°C.

(7)

$$\frac{dP}{dT} = \frac{LH}{T(v_g - v_f)} = \frac{LH}{T \cdot v_{fg}} \quad \text{—①}$$

$\ln P = A - \frac{B}{T}$

differentiating:

$$\frac{1}{P}\frac{dP}{dT} = 0 - B\left(-\frac{1}{T^2}\right)$$

$$\frac{1}{P}\frac{dP}{dT} = \frac{B}{T^2}$$

$$\frac{dP}{dT} = \frac{BP}{T^2} \quad \text{—②}$$

from ① & ②

$$\frac{BP}{T^2} = \frac{LH}{\cancel{T}\, v_{fg}}$$

$$\Rightarrow LH = \frac{BP \cdot v_{fg}}{v_{fg}\, T}$$

$$\Rightarrow h_{fg} = \frac{BP\, v_{fg}}{v_{fg}\, T}$$

$$\therefore \boxed{h_{fg} = \frac{P_{sat}\, v_{fg}\, B}{T_{sat}}} \quad (B)$$

(a)

(8)

Temp^r decrease & irreversible (a).

from Maxwell's eq^n:

(9) $\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$

$\frac{C_p}{C_v} = 1.044$ [illegible]

for ideal gas $PV = mRT$

$PV = RT$ (Take $m=1$).

$P = \frac{R}{V} \cdot T$

This is of the form $y = cx$.

$dy/dx = c$

$\therefore \left(\frac{\partial P}{\partial T}\right)_V = R/V$ (b).

(13) — (c)
(12) — (a)
(14) — (d)
(15) — (b)
(13)

(10) — (c)

(11) $(\Delta S) = -m_1R_1 \ln \frac{P_1}{P} - m_2R_2 \ln \frac{P_2}{P}$

$(\Delta S) = -m_1R_1 \ln x_1 - m_2R_2 \ln x_2$

$x_1 = \frac{n_1}{n_1+n_2} = \frac{1}{1+4} = 0.2$

$x_2 = \frac{n_2}{n_1+n_2} = 0.8$

$PV = mRT = n\bar{R}T$
$mR\cancel{T} = n\bar{R}\cancel{T}$
$mR = n\bar{R}$
$m_1R_1 = n_1\bar{R}$
$m_2R_2 = n_2\bar{R}$

$(\Delta S) = -n_1\bar{R}\ln x_1 - n_2\bar{R}\ln x_2$

$= -\bar{R}\left[1\times \ln(0.2) + 4\times \ln(0.8)\right]$

$= -8.314 \times \left[1\times \ln 0.2 + 4 \ln 0.8\right] = 20.8$

⑰ $C_p - C_v = \frac{TV\beta^2}{K_T}$

$$C_p - C_v = \frac{(273+25) \times 0.000114 \times (5 \times 10^{-5})^2}{8.6 \times 10^{-12}}$$

$\therefore C_p - C_v = 9.8$

Again, $\frac{C_p}{C_v} = 1.024$

$\Rightarrow C_v = \frac{C_p}{1.024}$

$\therefore C_p - \frac{C_p}{1.024} = 9.87$

$\therefore C_p = \frac{9.87}{0.024} = \underline{\underline{421.3}}$

Chapter - 3

Work Book

① — ⓓ
② — ⓐ
③ — ⓐ
④ — ⓓ
⑤ — ⓒ
⑥ — ⓐ
⑦ — ⓐ
⑧ — ⓒ
⑨ — ⓑ
⑩ — ⓒ
⑪ — ⓓ
⑫ — ⓒ
⑬ — ⓑ
⑭ — ⓓ
⑮ — ⓐ
⑯ — ⓓ
⑰ — ⓓ
⑱ — ⓓ
⑲ — ⓓ
⑳ — ⓐ
㉑ — ⓐ
㉒ — ⓐ
㉓ — ⓐ
㉔ — ⓒ
㉕ — ⓐ
㉖ — ⓒ
㉗ — ⓑ
㉘ — ⓐ
㉙ — ⓑ
㉚ — ⓒ
㉛ — ⓐ
㉜ —

② $h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2}$

$C_p = 1.005$ kJ/kg K

$P_1 = 100$ kPa, $T_1 = 500$ K, $C_1 = 150$ m/s, $T_2 = 510$ K.

$$h_1 + \frac{C_1^2}{2} + \cancelto{0}{Z_1 g} + \cancelto{0}{q} = h_2 + \frac{C_2^2}{2} + \cancelto{0}{Z_2 g} + \cancelto{0}{w}$$

$$h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2}.$$

$$h_1 + \frac{C_1^2}{2000} = h_2 + \frac{C_2^2}{2000}$$

$$\Rightarrow C_pT_1 + \frac{C_1^2}{2000} = C_pT_2 + \frac{C_2^2}{2000}$$

$$= 1.0005 \times 500 + \frac{(150)^2}{2000} = 1.005 \times 510 + \frac{C_2^2}{2000}$$

$\Rightarrow C_2 = 48.98 \simeq 49$ m/s. ④ diffuser ∵ final velocity is less than initial velocity.

③ $\dot{m} = 1$ kg/s, $T_1 = 800$ K, $P_1 = 200$ kPa, $C_1 = 50$ m/s.
$T_2 = 650$ K, $P_2 = 100$ kPa, $q = -25$ kW

$C_p = 1.005$ kJ/kg K.

$$h_1 + \frac{C_1^2}{2} + \cancelto{0}{Z_1 g} + q = h_2 + \frac{C_2^2}{2} + \cancelto{0}{Z_2 g} + \cancelto{0}{W}$$

$$\Rightarrow C_pT_1 + \frac{C_1^2}{2000} + q = C_pT_2 + \frac{C_2^2}{2000}$$

1 kW
$q = \frac{Q}{m} = \frac{25 \text{ kJ/s}}{1 \text{ kg/sec}}$
$= -25$ kJ/kg.

$$\Rightarrow 1.005 \times 800 + \frac{(50)^2}{2000} - 25 = 1.005 \times 650 + \frac{(C_2)^2}{2000}$$

$\Rightarrow C_2 = 503.98 \simeq \underline{504 \text{ m/s}}$. ④

⑨ $\delta Q = dE + \delta W$

$dE = \delta Q - \delta W$

(Properties are exact differentials).

⇒* OPEN SYSTEM WORK *⇐

Assumptions:

① Steady flow

② Neglect K.E and P.E changes

③ Reversible

$$h_1 + \cancelto{0}{\frac{C_1^2}{2}} + \cancelto{0}{Z_1 g} + q = h_2 + \cancelto{0}{\frac{C_2^2}{2}} + \cancelto{0}{Z_2 g} + w$$

$h_1 + q = h_2 + w$

or, $q = (h_2 - h_1) + w$

or, $q = dh + w$ — ①

Also

$\frac{dQ}{T} = ds$

$dQ = Tds$

$dq = Tds$

$q = Tds$. — ②

from ① & ②

$Tds = dh + w$ — ③

we also know $Tds = dh - vdP$ — ④

from ③ & ④

$\boxed{w = -vdP}$ open system work.

⑥ $P_1 = 400\,kPa$, $T_1 = 900K$, $C_1 = 0$.

$P_2 = 100\,kPa$,

~~PV = mRT~~ flow is ideal means reversible.

$$h_1 + \frac{C_1^2}{2} + \cancel{z_1 g}^{\to 0} + \cancel{q}^{\to 0} = h_2 + \frac{C_2^2}{2} + \cancel{z_2 g}^{\to 0} + \cancel{w}^{\to 0}.$$

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

(173)

$$\Rightarrow T_2 = 900 \times \left(\frac{100}{400}\right)^{\frac{0.4}{1.4}} = 605.6K$$

$$h_1 + \cancel{\frac{C_1^2}{2000}}^{\to 0} = h_2 + \frac{C_2^2}{2000}$$

$$\Rightarrow C_p T_1 + [illegible] = \gamma C_p T_2 + [illegible] \frac{C_2^2}{2000}.$$

$$\Rightarrow 1.005 \times 900 = 1.005 \times 605.6 + \frac{C_2^2}{2000}$$

$$\Rightarrow \underline{\underline{C_2 = 769\ m/s.}}$$ ⓐ

⑧ $m = 1\,kg/s$.

$P_1 = 0.1\,MPa = 100\,kPa$ $P_2 = 1000\,kPa$

$T_1 = 300K$.

$$T_2 = T_1 \times \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$= 300 \times \left(\frac{1000}{1000}\right)^{\frac{0.4}{1.4}}$$

$$\therefore \underline{T_2 = 579.2\,K}.$$

$$W = h_2 - h_1$$

$$= C_p(T_2 - T_1)$$

$$= 1.005(579.2 - 300) = 280.6\ kJ/kg$$

$$\therefore P = (280 \times 6 \times 1) = \underline{280.6\,KW}. \text{ or } \underline{280\,kW}$$

⑪ - ⓒ, ⑫ ⓓ

$\delta Q = dU + \delta W$

$\underline{10 = 30 - + 150}$

$\underline{50 = 30 - 80.}$

(176)

[illegible] ⑩ $m = 1\,kg.$

$dQ = dU + dW$

$0 = dU + dW$

$0 = mc_v(T_2 - T_1) + dW$

$dW = -mc_v(T_2 - T_1)$

$dW = mc_v(T_1 - T_2)$

$= 1 \times c_v(T_1 - T_2) = \underline{c_v(T_1 - T_2)}.$ ⓒ

⑪

1-2	2-3	3-1
$V = c;$	$T = c$	$P = c.$
$Q_{12} = 50$	$W_{23} = 500\,kj$	$W_{31} = -200\,kj$
$\because V = c$	for $T = c$	$Q_{31} = ?$
$\therefore W_{12} = 0.$	$W = Q$	$dU_{31} = ?$
	$\therefore Q_{23} = 500\,kj$	

for a cycle,

$\sum Q = \sum W$

$\Rightarrow Q_{12} + Q_{23} + Q_{31} = W_{12} + W_{23} + W_{31}$

$\Rightarrow 50 + 500 + Q_{31} = 0 + 500 - 200$

$\Rightarrow \boxed{Q_{31} = -250\,kj}$

$\delta Q = \delta U + \delta W$

$\Rightarrow -250 = dU + 200$

$\Rightarrow \boxed{\delta U_{31} = -50\,kj}$

ⓓ

⑭ $E = (25 + 0.25\,t)$ kJ ⑰

$dQ = dE + dW$

$-50 = 30 - 80$

$dQ = dE + dW$

$E = 25 + 0.25\,t$

$$\frac{dQ}{dt} = \frac{dE}{dt} + \frac{dW}{dt}$$

$$\frac{dE}{dt} = 0.25$$

$$\frac{dW}{dt} = 0.75$$

$$\frac{dQ}{dt} = 0.25 + 0.75 = 1.$$

(d)

⑮

1-2	2-3	3-1
$V = C_1$	$P = C$	$Q_{31} = 0.$
$Q_{12} = 170$ kJ	$Q_{23} = -180$ kJ	
$W_{12} = 0$	$W_{23} = -40$ kJ	$\therefore U_1 = 100$ kJ
$u_{12} = 100$ kJ	$dQ_{23} = du_{23} + dW_{23}$	$U_2 = 270$ kJ
$dQ_{12} = du_{12} + dW_{12}$	$\Rightarrow -180 = (U_3 - U_2) + 40$	$U_3 = 130$ kJ
$\Rightarrow 170 = U_2 - U_1 + 0$	$\Rightarrow -140 = (U_3 - 270)$	$\therefore$ (a)
$\Rightarrow U_2 = 170 + 100 = 270$ kJ	$\Rightarrow U_3 = 130$ kJ	

⑯ $P_1 = 10$ bar, $P_2 = 1$ bar.

In throttling $h_1 = h_2$

or, $u_1 + P_1 v_1 = u_2 + P_2 v_2$

$\Rightarrow (u_2 - u_1) = P_1 v_1 - P_2 v_2$

$= (1000 \times 0.5) - (100 \times 2)$

$= 300$ kJ/kg. (d)

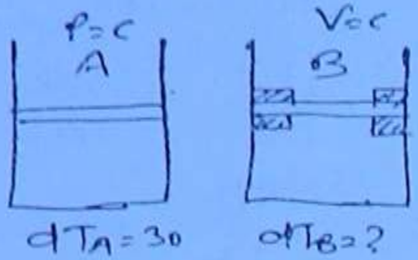
⑰ $dQ = dU + dW$

Question:- $\frac{dU}{Q_p} = ?$ $\quad \frac{dU}{Q_p} = \frac{dU}{dh} = \frac{m c_v dT}{m c_p dT}$ ($\because$ ideal gas)

$$= \frac{c_v}{c_p} = \frac{1}{\gamma} = \left(\frac{5}{7}\right)$$ (d).

⑱

A	B
$T_1 = 300K$	$T_1 = 300K.$
$P = C$	$V = C$
$Q_A = Q_B$	$Q_B = Q_A$
$T_2 = 330K.$	$T_2 = ?$



$(Q_A = Q_B).$

~~$Q_A = dh = mc_p dT$~~

$(Q_A)_P = dh = mc_p dT$

$(Q_B)_V = mc_v dT$ (128)

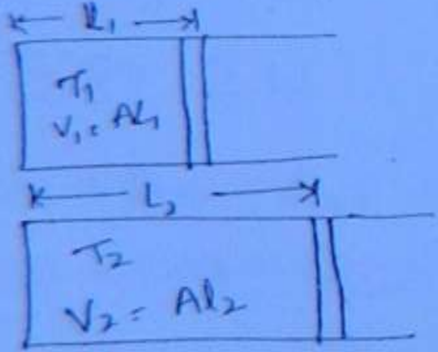
Now, $\cancel{m_A} c_p dT_A = \cancel{m_B} c_v dT_B$

$$\Rightarrow dT_B = \frac{c_p}{c_v} \cdot dT_A$$

$$= \gamma \times 30$$

$$\therefore \boxed{dT_B = 1.4 \times 30 = 42K.}$$ ⓓ.

⑲



for adiabatic process:-

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$\left(\frac{T_1}{T_2}\right) = \left(\frac{V_2}{V_1}\right)^{\gamma-1}$$

$$= \left(\frac{\cancel{A}L_2}{\cancel{A}L_1}\right)^{\frac{5}{3}-1}$$

$$= \left(\frac{L_2}{L_1}\right)^{2/3}$$ ⓓ

(20) $\delta Q = dU + \delta W$

$\Rightarrow \delta Q = dU \quad (\because \delta W = 0).$

for ideal gas $U \rightarrow f(T)$.

$\therefore \delta Q < 0$ mean $dU < 0$.

$\therefore$ Tempr will decrease.

(21) $\Sigma Q = 5\,kJ$

$\Sigma Q = \Sigma W = 5\,kJ$

$\therefore W_{AB} + W_{BC} + W_{CA} = 5$

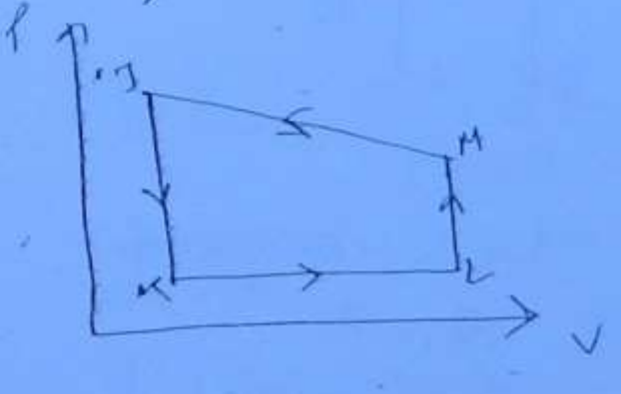
$\Rightarrow W_{AB} + 0 + W_{CA} = 5$

Now, $W_{AB} =$ Area under curve

$= (10 \times 1) = 10\,kJ$

$\therefore W_{CA} = 5 - 10 = -5\,kJ$ (a)

(22)



$J \rightarrow K \Rightarrow W = 0$

& $Q < 0$ ($\because$ heat rej.)

$P\cancel{V} = \cancel{mR}T$

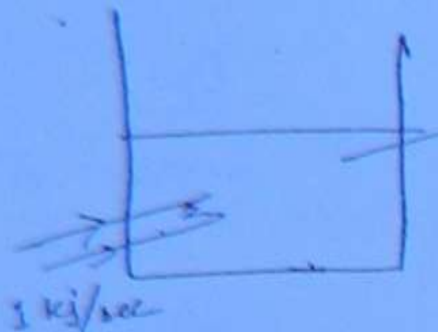
$P \propto T$

$P\downarrow \rightarrow T\downarrow$

$L - M \Rightarrow W = 0.$

(a)

㉓



$\rightarrow 160\,J/sec = 0.16\,kj/sec$

Net accumulation $= 1 - 0.16$

$= 0.84\,kj/sec$

for 't' seconds

$Q = 0.84\,t\ kj$

$Q = mc(\Delta T)$

$\therefore\ 0.84t = mc(\Delta T)$

$\Rightarrow 0.84t = 2 \times 4.2(77-27)$

$\therefore\ t = 500$ seconds

$\therefore\ t = \underline{8\ min.\ 40\ sec.}$ ⓐ

(180)

㉕ – ⓐ

㉖ – ⓒ

㉔

1-2 discharging

$Q_{12} = -250\,kcal.$

$W_{12} = 0.$

$1\,cal = 4.18\,J$

$1\,kcal = 4.18\,kj$

$\therefore\ Q_{12} = -250 \times 4.18\,kj$

$= -1045\,kj$

2-1

charging

$W_{21} = -0.53\,kW\,hr.$

$Q_{21} = ?$

$= -0.53\,\frac{kj}{sec} \times 3600\,sec$

$W_{21} = -1908\,kj$

$\Sigma Q = \Sigma W$

$Q_{12} + Q_{21} = W_{12} + W_{21}$

$-1045 + Q_{21} = 0 + (-1908)$

$\therefore\ \boxed{Q_{21} = -863\,kj}$ — ⓒ

(27) $dQ = dU + dW$

$$\frac{dQ}{dt} = \frac{dU}{dt} + \frac{dW}{dt}$$

$$\frac{dW}{dt} = -260 \text{ J/sec}$$

181

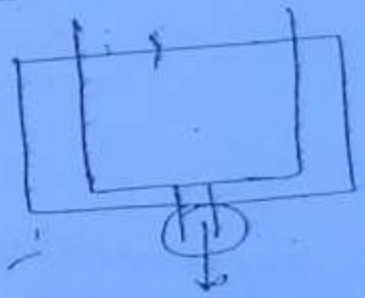
$$\frac{dQ}{dt} = -10 \text{ J/sec}$$

$$-10 = -260 + \frac{dU}{dt}$$

$$\therefore \frac{dU}{dt} = 250 \text{ J/sec}$$

$$\frac{250 \text{ J}}{\text{sec}} \times 3600 = 900 \times 10^3 \text{ J} = \underline{\underline{900 \text{ kJ}}}.$$ (b)

(28)

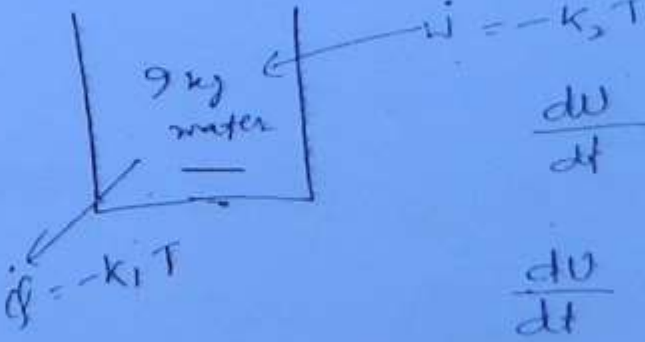


$$\left(\frac{dU}{dt}\right)_{cv} = \cancelto{0}{\dot m_i h_i} + \cancelto{0}{\dot Q} - \dot m_e h_e - \cancelto{0}{\dot W}$$

$$\left(\frac{dU}{dt}\right)_{cv} = -\dot m_e h_e$$

$$\frac{d}{dt}(mu) = -\dot m_e h_e$$ —— (a).

(29)



$$\frac{dU}{dt} = \cancelto{0}{\dot m_i h_i} + \dot Q - \cancelto{0}{\dot m_e h_e} - \dot W$$

(∵ no mass entering, no mass leaving)

$$\frac{dU}{dt} = \dot Q - \dot W = -K_1 T - (-K_2 T)$$

$$\frac{dU}{dt} = (K_2 - K_1) T$$

$$\frac{dU}{dt} \cdot \frac{dT}{dT} = (K_2 - K_1)T$$

$$\frac{dU}{dT} \cdot \frac{dT}{dt} = (K_2 - K_1)T$$

from ideal gas eq^n, $dU = c\,dT$ or $\frac{dU}{dT} = c$.

$$\therefore\ c \cdot \frac{dT}{dt} = (K_2 - K_1)T.$$

or $$\frac{dT}{T} = \frac{(K_2 - K_1)}{c} \cdot dt$$

<u>Integrating :-</u>

$$\ln T = \frac{(K_2 - K_1)}{c} \cdot t$$

$$T = e^{\left[\left(\frac{K_2 - K}{c}\right) \cdot t\right]}$$

If $K_2 > K_1$, T will increase exponentially

If $K_2 < K_1$, T will decrease exponentially.

5) Use steam table and find enthalpy at 1 MPa & 400°C and then calculate.

Ans is (c) - 0.612

(31) $P_1 = 0.7$ bar $= 70$ kPa.

$t_1 = 57^\circ C = 330K.$

$C_1 = 200$

$A_1 = 0.4$

$P_2 = 1$ bar.

$A_2 = 0.6$

(183)

$$\overset{\circ}{m} = \rho_1 A_1 C_1 = \rho_2 A_2 C_2$$

$$\overset{\circ}{m} = \rho_1 A_1 C_1 .$$

∵ Ideal gas, ... $PV = mRT$

$$\frac{m}{V} = \frac{P}{RT}$$

$$\rho = P/RT.$$

$$\therefore \rho_1 = \frac{P_1}{R_1 T_1} = \frac{70}{0.287 \times 330} = 0.739.$$

$$\therefore \overset{\circ}{m} = 0.739 \times 0.4 \times 200$$

$$\therefore \boxed{\overset{\circ}{m} = 59.1 \text{ kg/sec.}}$$ CM (a)

(32) from steady flow equation :-

$$h_1 + \frac{C_1^2}{2000} + \cancel{zg}^{\,0} + \cancel{q}^{\,0} = h_2 + \frac{C_2^2}{2000} + \cancel{zg}^{\,0} + \cancel{w}^{\,0}$$

$$\therefore h_1 + \frac{C_1^2}{2000} = h_2 + \frac{C_2^2}{2000}$$

$$\text{or } C_p T_1 + \frac{C_1^2}{2000} = C_p T_2 + \frac{C_2^2}{2000} \quad \text{—— ①}$$

Also $\overrightarrow{m} = \frac{P}{RT_2} A_2 C_2$ $\quad (\rho_2 A_2 C_2).$

$$59.1 = \frac{100}{0.287 T_2} \times 0.6 \times C_2$$

$$\rightarrow C_2 = 0.283 T_2 \quad \text{—— ②}$$

Putting ② in ①, we can get answer

-: I.C. ENGINE :-

B. Goswami

An I.C. Engine compraises of a piston cylinder arrangement where the combustion takes place internaly for attaining the useful work output. The different important cycles that are involved for attaining useful work output are :-

1> The otto cycle or the constant volume cycle. (185)

2) The Diesel cycle or the constant pressure cycle.

3) The Dual combustion cycle

The common processes that are involved for the above cycles are :-

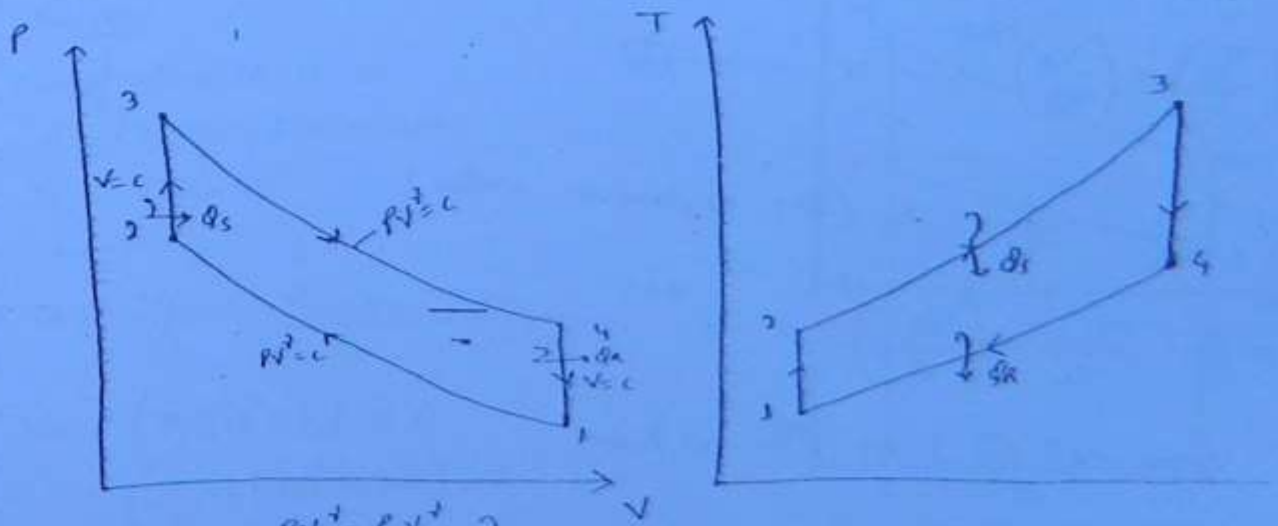
a) Isentropic expansion.

b) Heat rejection at constant volume.

c) Isentropic compression.

The different cycles are different from one another because of heat addition taking place differently.

⟹* THE OTTO CYCLE OR THE CONSTANT VOLUME CYCLE :-



$$\left.\begin{aligned} P_1V_1^{\gamma} &= P_2V_2^{\gamma} \\ \frac{P_1V_1}{T_1} &= \frac{P_2V_2}{T_2} \end{aligned}\right\} \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_2}{T_1}$$

$$dS = mC_v \ln\frac{T_2}{T_1}$$ (Change in entropy when volume is constant)

If T_2 is greater, dS will greater than 0.

⇒* Derivation of efficiency of otto cycle :-

~~$\eta = \frac{WD}{HS} = \frac{HS-HR}{HS} = 1-\frac{HR}{HS}$~~

~~$\eta = \frac{WD}{HA} = \frac{HA-HR}{HA} = 1-\frac{HR}{HA}$~~

~~$HA =$~~

We know that the efficiency of the cycle

$$\eta_v = \frac{WD}{HA} = \frac{HA-HR}{HA} = 1-\frac{HR}{HA}$$

$$HA = mC_v(T_3 - T_2)$$

$$HR = mC_v(T_4 - T_1)$$

$$\eta_v = 1 - \frac{mC_v(T_4-T_1)}{mC_v(T_3-T_2)} \quad \text{—— ⓐ}$$

Let $\frac{V_1}{V_2} = r$; (the compression ratio)

we know that for an adiabatic process;

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = r^{\gamma-1} \quad \text{—— ⓑ}$$

But, $\frac{V_4}{V_3} = \frac{V_1}{V_2} = r$; (the expansion ratio)

& $$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} = r^{\gamma-1} \quad \text{—— ⓒ}$$

from eqⁿ ⓑ & eqⁿ ⓒ, we have

$$\frac{T_2}{T_1} = \frac{T_3}{T_4}$$

$$\therefore \frac{T_4}{T_3} = \frac{T_1}{T_2}$$

from componendo and dividendo, we have :-

$$\frac{a}{b} = \frac{c}{d} = \frac{a-c}{b-d}$$

$$\therefore \frac{T_4}{T_3} = \frac{T_1}{T_2} = \frac{T_4-T_1}{T_3-T_2} \quad \text{(from algebra)} \text{ — ⓓ}$$

Thus, we have :-

$$\eta_v = 1 - \frac{T_4-T_1}{T_3-T_2} = 1 - \frac{T_1}{T_2} \quad (\text{from eq}^n \text{ (d)})$$

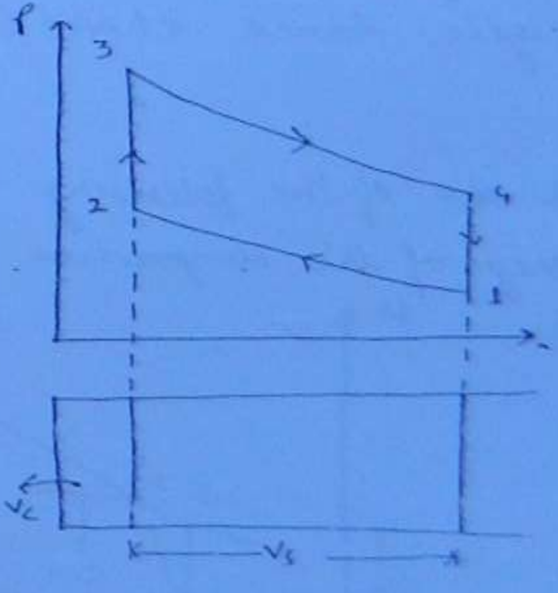
$$\text{or, } \boxed{\eta_v = 1-\left(\frac{1}{r}\right)^{\gamma-1}} \text{ — ①} \quad (\text{from equation (b)}).$$

Problem

ESE-2009.

Q:- The bore and stroke of the cylinder of a ~~o~~ 6-cylinder engine working on an otto cycle are 17 cm and 30 cm respectively. Total clearance volume is 9225 cm³. Find the compression ratio.

Sol^n:- Given:



No. of cylinders $n = 6$

$d = 17$ cm, $L = 30$ cm

$V_c = 9225 \text{ cm}^3$

$r = ?$

$$r = \frac{V_1}{V_2} = \frac{V_s + V_c}{V_c}$$

$$V_c = \left(\frac{\pi}{4} d^2 \times L\right) \times 6$$

$$= \left(\frac{\pi}{4} \times (17)^2 \times 30\right) \times 6$$

$$\therefore V_s = 40856.4 \text{ cm}^3$$

Shafi
Shaf

Now,

$$r = \frac{V_s + V_c}{V_c}$$

$$= \frac{40856.4 + 9225}{9225} \Rightarrow$$

$$\therefore \boxed{r = 5.43} \text{ (Ans).}$$

E-2008

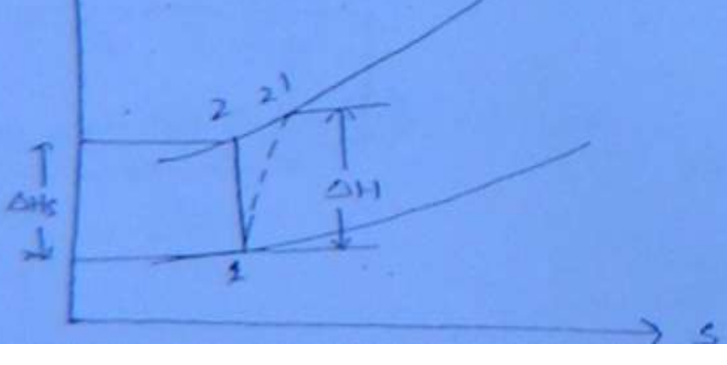
Q- Which gas power cycle consists of four processes & during which work done alone is transferred during two processes and heat added alone during two ~~at~~ processes?

- ⓐ Atkinson cycle
- ⓑ Carnot cycle
- ⓒ Diesel cycle
- ✓ⓓ Otto cycle. (Ans).

During compression and expansion process, there is no heat transfer for an otto cycle, there is existence of work done. The heat addition and heat rejection process is at constant volume. he work done during constant volume process is zero (0). As, iere is only one type of transfer during each process of e otto cycle, hence, choice (d) is correct.

-2005

Which one of the following expressions, the isentropic efficiency of the compression process is indicated in the figure.



✓ ⓐ $\eta = \dfrac{\Delta H_s}{\Delta H}$

ⓑ $\eta = \dfrac{\Delta H}{\Delta H_s}$

ⓒ $\eta = \dfrac{\Delta H - \Delta H_s}{\Delta H}$

ⓓ $\eta = \dfrac{\Delta H - \Delta H_s}{\Delta H_s}$

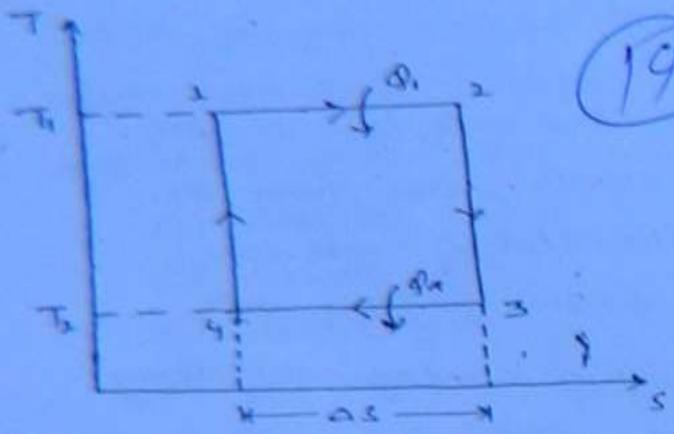
(189)

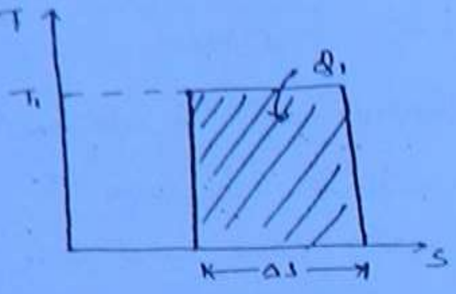
When the compression is without friction, the work done is 'ΔH_s' (see figure). When friction is taken in account, then from the figure, the work done will be 'ΔH'. The ratio of work done without friction to the workdone ~~with~~ when friction is taken into account is defined as compression efficiency or isentropic efficiency. This efficiency for the given figure is $\eta = \dfrac{\Delta H_s}{\Delta H}$ (choice ⓐ is correct).

-: AVAILABLE ENERGY, AVAILABILITY & IRREVERSIBILITY :-

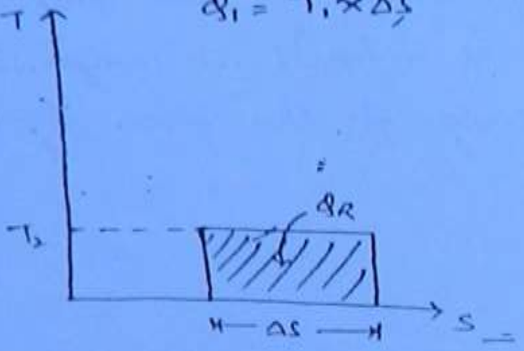
Available Energy (AE) :-

→ The maximum possible amount of work that can be obtained in a cycle is known as available energy.





$Q_1 = T_1 \times \Delta S$



$Q_R = T_2 (\Delta S)$

$$\eta = W/Q_1$$

$$\eta_{max} = \frac{W_{max}}{Q_1}$$

also, $\eta_{max} = 1 - \frac{T_2}{T_1}$

$$\Rightarrow \frac{W_{max}}{Q_1} = 1 - \frac{T_2}{T_1}$$

$$\Rightarrow \boxed{W_{max} = Q_1 (1 - T_2/T_1)}$$

For a given source T_1 and for a given input Q_1 this work is further maximum when T_2 is minimum. The lowest possible tempr of heat rejection is that of surroundings ($T_2 = T_0$). Therefore,

$$\boxed{AE = W_{max} = Q_1 \left(1 - \frac{T_0}{T_1}\right)}$$

Now, by process graph:-

$$Q_1 = T_1 \Delta S$$

$$\Rightarrow \frac{Q_1}{T_1} = \Delta S$$

Now, $AE = Q_1\left(1 - \frac{T_0}{T_1}\right)$

$$= Q_1 - \frac{Q_1}{T_1} T_0$$

$$= Q_1 - T_0 \Delta S$$

$$\Rightarrow Q_1 = AE + T_0 \Delta S$$

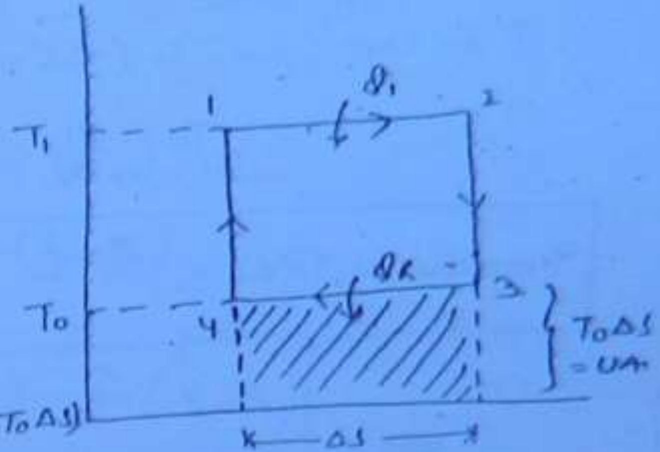
$$\boxed{Q_1 = AE + UAE}$$

Heat Rejection:-

$$Q_R = \underbrace{T_0 \Delta S}_{\text{min heat rejection}}$$

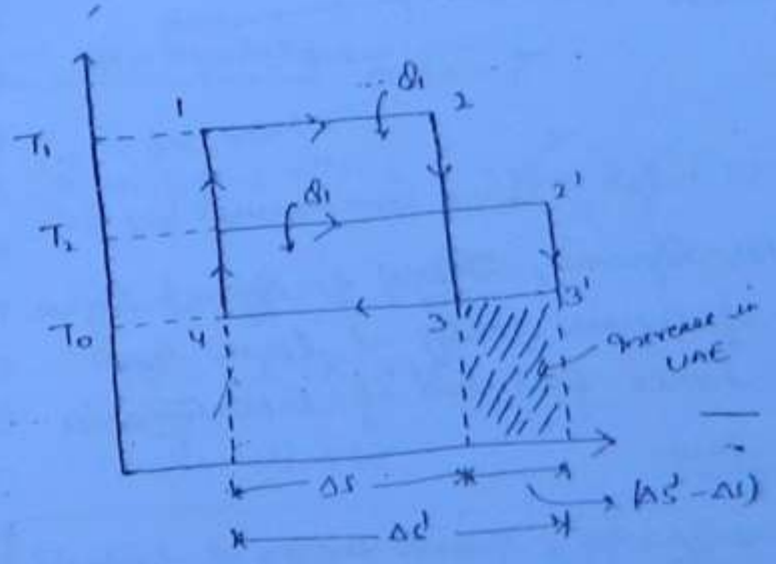
↓

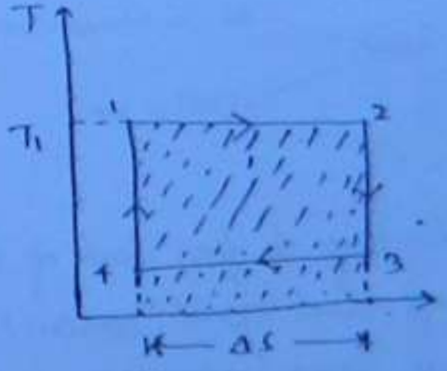
UAE



The minimum possible heat rejection ($T_0 \Delta S$) is known as unavailable energy (UAE) i.e. area below atmospheric temp. T_0 always represents unavailable energy.

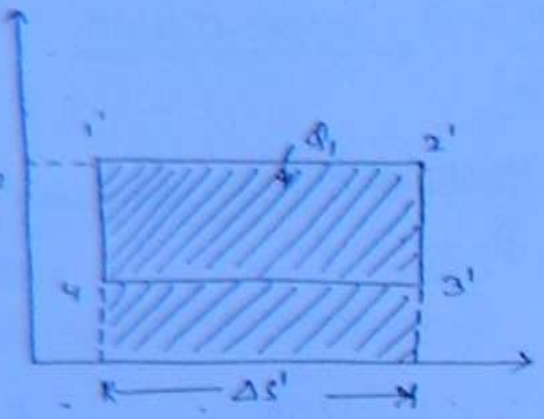
* Loss of Available Energy (Increase in unavailable energy) when heat is transferred through a finite temp:-





$$Q_1 = T_1 \Delta S$$

$$\Rightarrow \Delta S = \frac{Q_1}{T_1}$$



$$Q_1 = T_2 \times \Delta s'$$

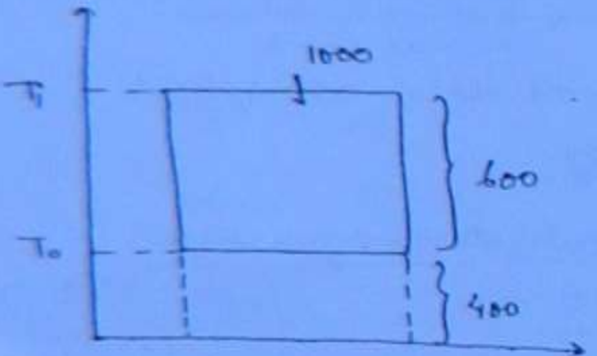
$$\Rightarrow \Delta s' = \frac{Q_1}{T_2}$$

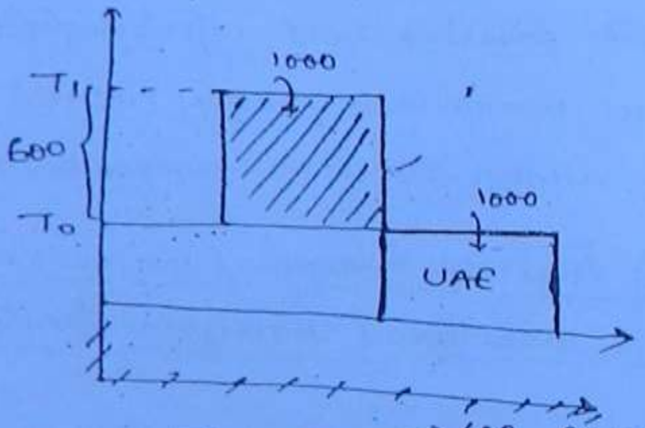
192

Now. $\uparrow$ in UAE $= T_0(\Delta s' - \Delta s)$

$$= T_0\left(\frac{Q_1}{T_2} - \frac{Q_1}{T_1}\right)$$

$$\boxed{\text{Increase in UAE} = Q_1 T_0 \left(\frac{T_1 - T_2}{T_1 T_2}\right)}$$





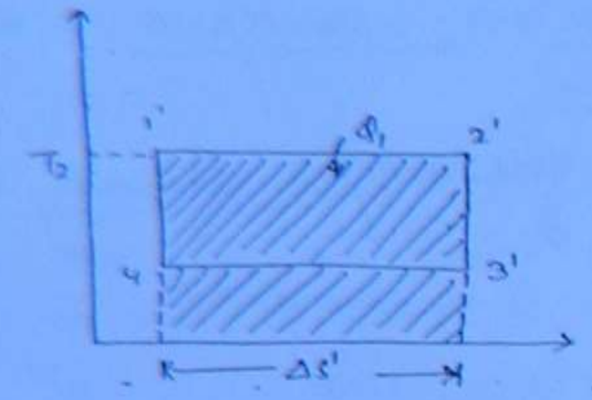
$Q_1 = 1000$ → AE = W_{max} = 600
$Q_1 = 1000$ → UAE = 400

$T_1 \rightarrow 1000 \rightarrow 600$ AE = W
$T_1 \rightarrow 1000 \rightarrow 400$ UAE

$T_0 \rightarrow 1000 \rightarrow$ UAE but $W = 0$.

** According to first law of thermodynamics, thermal energy at higher temp and equal amount of thermal energy at lower temp have same meaning and hence <u>first law</u> of thermodynamics is known as <u>quantitative law</u>.

** According to 2nd law of thermodynamics, thermal energy at higher temperature has greater significance when compared to the same amount of thermal energy at lower temp? because thermal energy at higher temp? is capable of producing more power and



$$Q_1 = T_2 \times \Delta s'$$

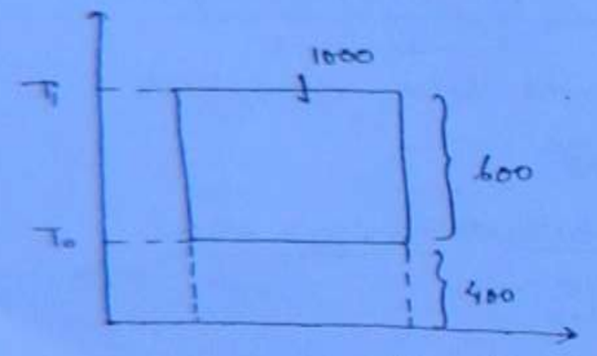
$$\Rightarrow \Delta s' = \frac{Q_1}{T_2}$$

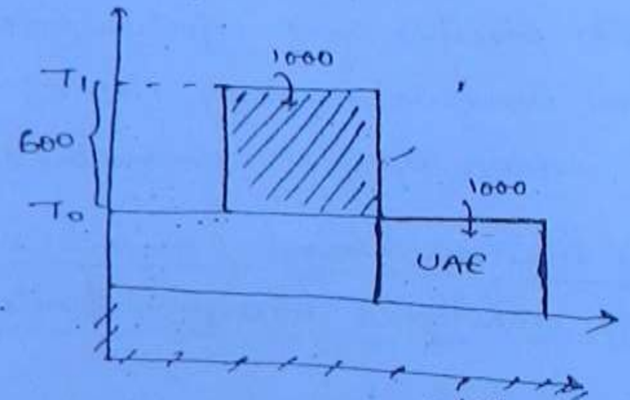
(192)

Now. ↑ in UAE $= T_0 (\Delta s' - \Delta s)$

$$= T_0 \left(\frac{Q_1}{T_2} - \frac{Q_1}{T_1} \right)$$

$$\boxed{\text{Increase in UAE} = Q_1 T_0 \left(\frac{T_1 - T_2}{T_1 T_2} \right)}$$





$Q_1 = 1000$ — AE $= W_{max} = 600$

— UAE $= 400$

$T_1 \rightarrow 1000 \rightarrow 600$ AE $= W$

$\rightarrow 400$ UAE

$T_0 \rightarrow 1000 \rightarrow$ UAE but $W = 0$.

** According to first law of thermodynamics, thermal energy at higher temp and equal amount of thermal energy at lower temp have same meaning and hence first law of thermodynamics is known as quantitative law.

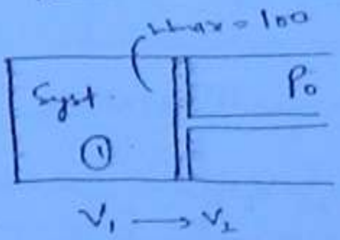
** According to 2nd law of thermodynamics, thermal energy at higher temperature has greater significance when compared to the same amount of thermal energy at lower temp. because thermal energy at higher temp. is capable of producing more power and

$\Rightarrow dW_{sys} = T_0 (\Delta S)_{sys} - dU_{syst}$ —— (from ①)

$$\Rightarrow W_{max} = T_0 (S_2 - S_1) - (U_2 - U_1)$$

$$= T_0 (S_2 - S_1) - U_2 + U_1$$

$$\Rightarrow \boxed{W_{max} = (U_1 - U_2) - T_0 (S_1 - S_2)}$$ — for closed system only.



$W_{max\,useful} = W_{max} - W_{atm}$

$W_{atm} = P_0 (V_2 - V_1)$

So, $$\boxed{W_{max\,useful} = (U_1 - U_2) - T_0 (S_1 - S_2) - P_0 (V_2 - V_1)}$$

$$= (U_1 - T_0 S_1 + P_0 V_1) - [U_2 - T_0 S_2 + P_0 V_2]$$

$W_{max\,useful} = \phi_1 - \phi_2$ where, $\phi_1 = U_1 - T_0 S_1 + P_0 V_1$

$$\Rightarrow \boxed{\phi = U - T_0 S + P_0 V}$$ — availability function for closed system.

Maxm work for open System :-

$$h_1 + \cancel{\frac{C_1^2}{2}} + \cancel{Z_1 g} + Q = h_2 + \cancel{\frac{C_2^2}{2}} + \cancel{Z_2 g} + W$$

$$\Rightarrow h_1 + Q = h_2 + W$$

$$Q = h_2 - h_1 + W$$

$$Q = dh + W$$

So, $$\boxed{W_{max} = h_1 - h_2 - T_0 (S_1 - S_2)}$$ ~~net change of availability~~

$$= (h_1 - T_0 S_1) - (h_2 - T_0 S_2)$$

$$\boxed{W_{max} = \phi_1 - \phi_2}$$ → change of availability.